Effective critical electric field for runaway electron generation

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Introduction The critical electric field for runaway electron (RE) generation is a classic result in plasma physics [1]. Recently, experimental evidence indicating that the electric field strength necessary for RE generation is in fact several times larger than the critical field has been presented [2]. Here we investigate factors that may influence the threshold field for experimental detection, and conclude that an elevated *effective* critical field is to be expected in many cases of practical interest.

We will consider the effects of the plasma temperature and radiation reaction due to synchrotron emission on the primary (Dreicer) RE generation. In order to study RE dynamics, we have developed an efficient finite-difference–spectral-method tool, CODE [3] (COllisional Distribution of Electrons), for solving the kinetic equation. CODE calculates the continuum electron distribution function in two-dimensional momentum-space in a homogeneous plasma. Both primary (Dreicer, hot-tail) and secondary RE generation mechanisms are included, and a Coulomb collision operator valid for arbitrary electron energies is used [4]. For this work, an operator for momentum loss and pitch-angle scattering due to synchrotron emission was implemented, based on the analytical results in Refs. [5, 6]. CODE can be run in a time-dependent mode, and can then include avalanche generation, but for this application primary generation is sufficient as avalanche generation requires an existing sub-population of highly energetic electrons (the avalanche growth rate is proportional to n_{RE}). As long as the runaway density is low, as is the case close to the critical field, CODE can be run in a time-independent mode where it is sufficient to solve a single system of equations to obtain a quasi-steady-state solution.

With the numerical electron distribution function from CODE, we are able to investigate the runaway generation for a wide range of plasma parameters. In the future, the numerical distribution could be used to for instance calculate the synchrotron radiation spectra of the REs [7], or to study wave-particle interactions [8, 9].

Temperature dependence The critical electric field E_c is the weakest field at which electron runaway is possible. It is given by $E_c = n_e e^3 \ln\Lambda/(4\pi\varepsilon_0 m_e c^2)$, where n_e and m_e are the electron density and mass, respectively [1]. At $E \gtrsim E_c$, however, only electrons already moving with approximately the speed of light may run away. In an infinite plasma in thermal equilibrium, some (in fact infinitely many) particles will have speeds arbitrarily close to that of light. In reality the plasma size is limited, and among the plasma particles there is an actual highest speed achieved (which may be significantly less than c). Thus, if the critical speed for RE generation at a given E-field is larger than this speed, no electrons will be able to run away. The width (in velocity space) of the Maxwellian distribution describing the particle speeds is determined by the temperature. This introduces an additional temperature dependence into the *effective* critical field, since the number of particles with speeds above any threshold speed v_c is temperature dependent. Mathematically, this can be understood from the runaway growth rate [1], which is exponentially small in $E/E_D = T_e/(m_e c^2) \cdot E/E_c$. There is thus an inherent temperature dependence in the primary runaway growth rate at a given value of E/E_c, and for significant RE production it is not enough to only require $E > E_c$.

In Fig. 1, a contour plot of the runaway growth rate of the quasi-steady-state electron distribution is shown as a function of the electron temperature and E/E_c. The figure was obtained using CODE, without secondary runaway generation or synchrotron losses. The figure shows that the fraction of the electron population that runs away in one second is less than 10^{-20} for all field strengths $E/E_c < 1.5$. Note that the figure essentially covers the whole temperature range of magnetic fusion plasma operation. In a plasma with $n_e \lesssim 10^{20}\,\mathrm{m}^{-3}$ and a volume of a few tens of m^3, essentially no runaways production (let alone detection) is thus to be expected. It is also clear that for lower temperatures, a stronger electric field is required for significant RE production. At $T_e = 10$ eV, a typical post-thermal-quench temperature in JET, significant RE production is expected only for $E/E_c \gtrsim 300$ (at $n_e = 5 \cdot 10^{19}$ m^3, $Z = 1.5$).

The white and black contours in Fig. 1 show the corresponding values of E/E_D. We can conclude that E/E_D must at least be larger than $1-2\%$ for significant runaway formation to occur. There are thus in practice two conditions that must be fulfilled: $E/E_c > 1$ and $E/E_D > k$, for some small k. Whichever is more restrictive depends on the temperature.

Momentum loss due to synchrotron emission Radiation reaction due to synchrotron emission is an important momentum loss mechanism for REs, as it introduces an additional drag force. The emitted synchrotron power scales strongly with the particle energy and pitch, and at multi-MeV energies it can effectively limit the runaway acceleration. In addition, although the collisional drag is monotonically decreasing for high

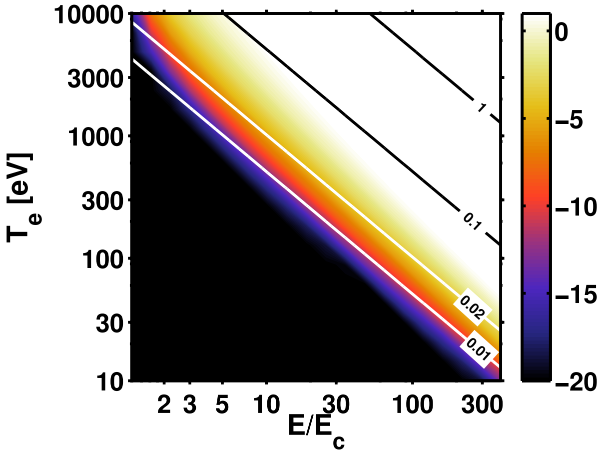

Figure 1: Runaway growth rate (particle fraction per second, $\log_{10}[d(n_{RE}/n_e)/dt]$*) as a function of temperature and electric field. The value of* E/E_D *is also shown (white and black contours).* $n_e = 5\cdot 10^{19}$ m^{-3} *and* $z_{eff} = 1.5$ *were used.*

momenta, the radiation reaction force increases with momentum, and the total friction therefore has a minimum at high but finite p. The friction force at this minimum is always higher than the collisional drag at infinite momentum, and synchrotron radiation reaction losses therefore lead to an increase in the critical electric field for RE generation.

To investigate the magnitude of this effect, a synchrotron loss operator [5, 6] was included in the kinetic equation in CODE. In the cylindrical limit, the radiation reaction is described by

$$S\{F\} = \hat{B}^2\left(-\gamma^{-1}\left[2+4p^2(1-\xi^2)\right]F - \gamma p\left(1-\xi^2\right)\frac{\partial F}{\partial p} + \frac{\xi}{\gamma}(1-\xi^2)\frac{\partial F}{\partial \xi}\right)$$

where $p = \gamma v/c$ is the normalized momentum, $\xi = p_{\|}/p$ is the cosine of the pitch angle, F is the normalized distribution used in CODE (see Ref. [3]) and $\hat{B}^2 = \nu_{ee}^{-1}\tau_r^{-1}$ is the square of a normalized magnetic field. What determines the strength of the synchrotron losses is thus the ratio of the collision time $1/\nu_{ee}$ to the radiation time scale $\tau_r(B) = 6\pi\varepsilon_0 m_e^3 c^5/(e^4 B^2)$ [5], where B is the magnetic field strength. For a given magnetic field, we expect the largest losses for high temperatures and low densities, since $\hat{B}^2 \propto T_e^{3/2} B^2/n_e$ (for constant $\ln\Lambda$).

Figure 2 shows the temperature (a) and density (b) dependence of the change in RE growth rate as a result of the synchrotron radiation reaction. From the figure, we conclude that the synchrotron losses can reduce the RE rate substantially for weak E-fields - by several orders of magnitude at high temperatures and low densities - and it is therefore essential to include the losses when considering near-critical RE dynamics. The sharp cut-off for weak fields is to be expected from the change in critical field associated with the inclusion of the synchrotron drag. For stronger electric fields, the effects are less pronounced (unless the density is very low). We note that in post-thermal-quench conditions (low T_e, high n_e), the effect of synchrotron losses are likely to be negligible, whereas in the case of

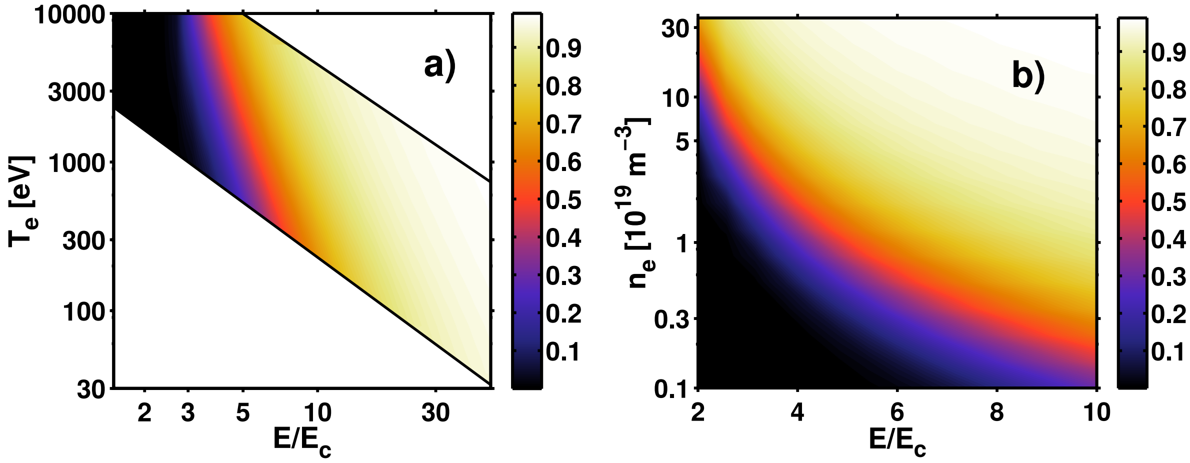

Figure 2: Contour plots of the a) temperature and b) density dependence of the ratio between the primary RE growth rate in CODE with and without synchrotron losses included. The parameters $B = 4$ T and $z_{eff} = 1.5$ and a) $n_e = 1\cdot 10^{19}$ m^{-3}, b) $T_e = 2$ keV were used. To ensure reliable results in a), the parameter region has been restricted as the growth rates are too small for low T_e and fields, and E/E_D approaches unity for high T_e and fields (cf. Fig. 1).

RE generation during the ramp-up phase (high T_e, low n_e) they can be substantial. There is thus a qualitative difference in the momentum space dynamics in these two cases - at least for near-critical electric fields - and conclusions drawn from studying REs generated during ramp-up are not necessarily applicable in post-disruption conditions.

Conclusions With numerical calculations, we have shown that the *effective* critical electric field, above which runaway generation is detected, is strongly dependent on the temperature of the plasma. In practice, $E/E_D > 1-2\%$ is required for substantial RE production. In addition, the drag due to synchrotron emission back reaction increases the critical field. For weak fields, the runaway growth rate can be reduced by orders of magnitude. Synchrotron losses must thus be taken into account when considering RE generation, especially when the temperature is high and the density low. In post-disruption plasmas the effects of synchrotron losses on the RE growth rate are likely to be negligible, whereas during ramp-up and flat-top, they can be substantial.

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