# Scaling Laws for Non-Stationary Biberman-Holstein Radiative Transfer in Plasmas

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**1. Introduction.** The Biberman-Holstein (B-H) equation is a fundamental tool for describing the radiative transfer (RT) in the resonance atomic/ionic spectral lines under condition of the complete redistribution over photon energy within spectral line width (i.e. full loss of memory) in the elementary act of absorption-emission by an atom/ion in plasmas and gases (see, e.g., [1], for astrophysics, and [2], for radiation losses and spectroscopy of fusion plasmas). The B-H model was applied to the hydrogen isotopes Lyman-alpha line RT in ITER and JET divertor [3] and the evaluation of plasma opacity impact upon current decay after disruptions in tokamaks [4].

Here we report on the scaling laws of the Green function [5] of the non-stationary B-H equation in an infinite homogeneous medium. It is shown that the Green function may be represented as an (auto-model) function of a single argument which depends on the scaling law for the excitation front propagation. This scaling is defined by a simple equation and describes essential non-locality of the B-H RT. The approximate auto-model solution is tested against exact numerical one for various spectral line shapes (Doppler, Lorentz, Voigt, Holtsmark) in a broad range of propagation time and distance from the source, including the asymptotic behavior far in advance of the propagation front and far behind it.

**2. Scaling laws for Green function.** The B-H equation is obtained from a system of equations for spectral intensity of resonance radiation and spatial density of excited atoms, $F(\mathbf{r},t)$. This system is reduced to a single equation for $F(\mathbf{r},t)$, which appears to be an integral equation, non-reducible to a differential diffusion-type equation ($A$ and $\sigma$ are the inverse lifetimes of excited atomic state with respect to spontaneous radiative decay and collisional quenching, respectively; $q$ is the non-radiative source of excited atoms):

$$\frac{\partial F(\mathbf{r},t)}{\partial t} = A\int_V G(|\mathbf{r}-\mathbf{r}_1|)F(\mathbf{r}_1,t)dV_1 - (A+\sigma)F(\mathbf{r},t)+q(\mathbf{r},t). \tag{1}$$

The kernel $G$ is determined by the (normalized) emission spectral line shape, $\varepsilon_\omega$, and the absorption coefficient $k_\omega$. In homogeneous media, $G$ depends on the distance between the points of emission and absorption of the quantum:

$$G(r) = -\frac{1}{4\pi r^2}\frac{dT(r)}{dr}, \qquad T(r) = \int_0^\infty \varepsilon_\omega \exp(-k_\omega r)\, d\omega. \tag{2}$$

The non-locality of the B-H RT demands special definition of the mean time, $\bar{t}(r)$, needed for a photon to pass the distance $r$ from a point instant source, $q(\mathbf{r},t) = \delta(\mathbf{r}-\mathbf{r}_0)\delta(t-t_0)$. The respective scalings for various line broadening mechanisms strongly deviate from the diffusion law (see [5, 2]). For Doppler and Lorentz line shapes, one has $\bar{t}(\rho) \approx 1/[AT_{as}(\rho)]$ [6], where $\rho \equiv k(\omega_0)r$, and $T_{as}(\rho)$ is the asymptotic of the Holstein functional $T$ at ρ>>1.

Our numerical analysis of Green function [5] for various line shapes shows that the scaling defined by the equation

$$At\,T(\rho) = 1\,, \tag{3}$$

gives good approximation for the time moment when $F(r,t)$ attains its maximum value at the distance $r$ from the source (Fig. 1). We use Eq. (3) as a definition of the effective front propagation, $\rho_{fr}(t)$.

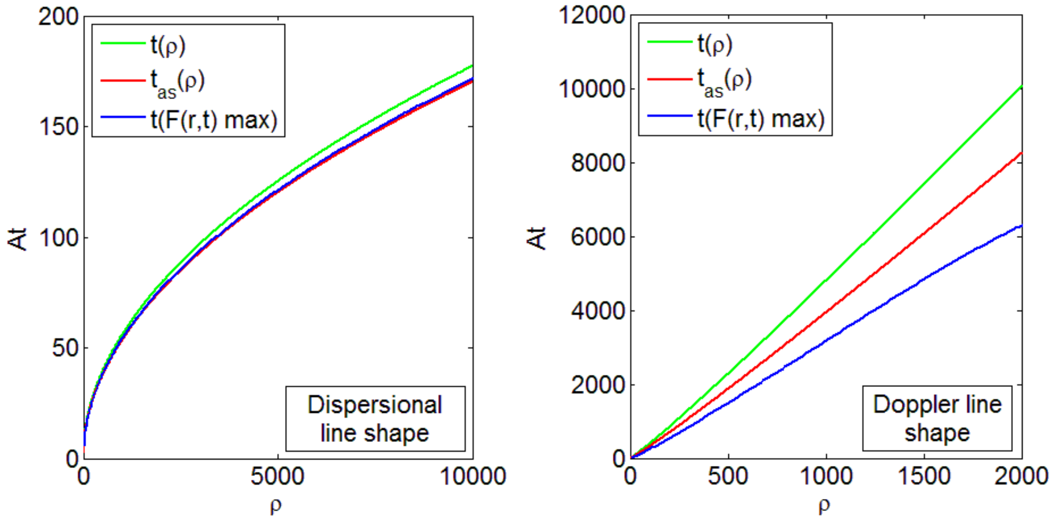



Fig. 1. Time moment when $F(r,t)$ attains its maximum value at the distance $r$ from the source (blue curve) and its approximations with Eq. (3) for $A$ = 1 (green curve) and $A = A_{as}$ (red curve) [6] ($(A_{as})^{-1}$ = 0.96 for Lorentz (dispersional) line shape and $(A_{as})^{-1}$ = 0.82 for Doppler line shape).

For a short time, $A^{-1} \ll t \ll t_{fr}(\rho)$ (or, equivalently, far in advance of front propagation coming at the distance $r$, $\rho \gg \rho_{fr}(t) \gg 1$), one has $F \approx t\,G(\rho)$ [5] that corresponds to the direct excitation of distant atoms by the photons in the far wings of the spectral line shape (i.e. by the Levy flights).

The Green function far behind the propagation front, $\rho \ll \rho_{fr}(t)$, or equivalently $t \gg t_{fr}(\rho) \gg A^{-1}$, may be estimated assuming the local uniformity of the excitation due to the fast exchange of atoms in the core of the spectral line shape. This gives a quasi-plateau:

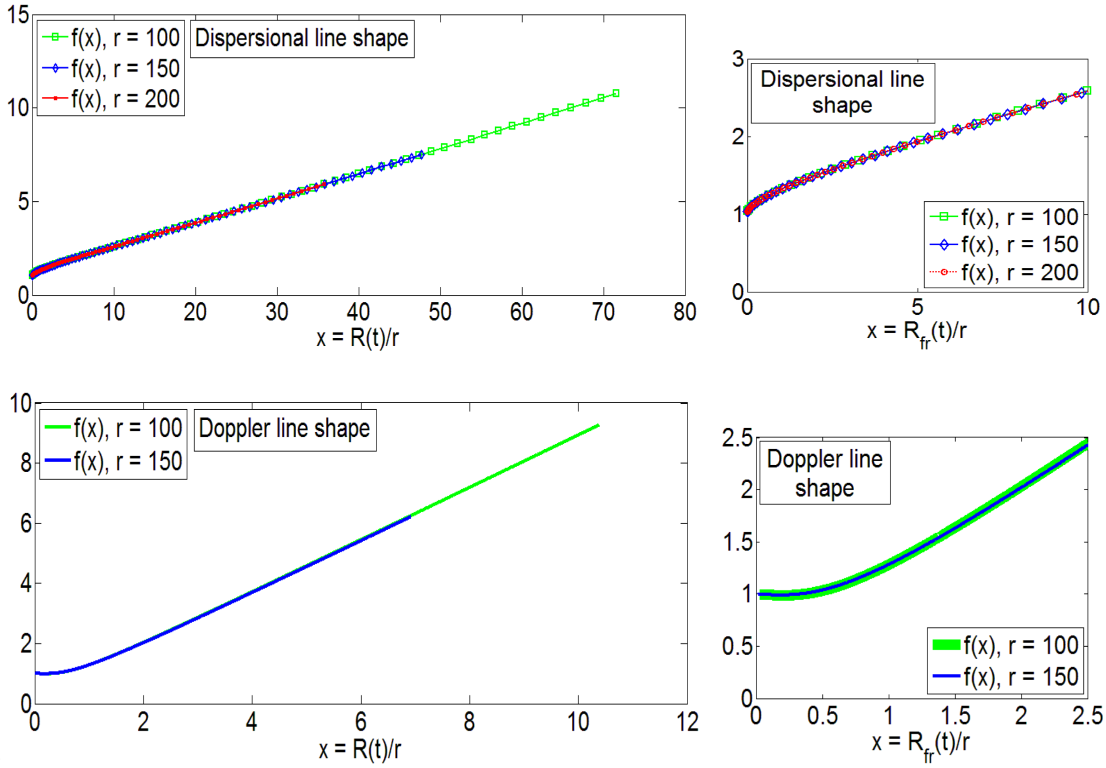
$$F(r,t) \approx \frac{1}{\frac{4}{3}\pi\left(r_{fr}(t)\right)^{3}} \eta\left(r_{fr}(t)-r\right) \tag{4}$$

Comparison of Eq. (4) with numerical calculations of the exact Green function [5] proves this asymptotic to give a good scaling for time dependence for various line shapes, with some deviation of numerical constant.

The success of the scaling (4) suggests an analysis of the Green function [5] with respect to the following auto-model representation:

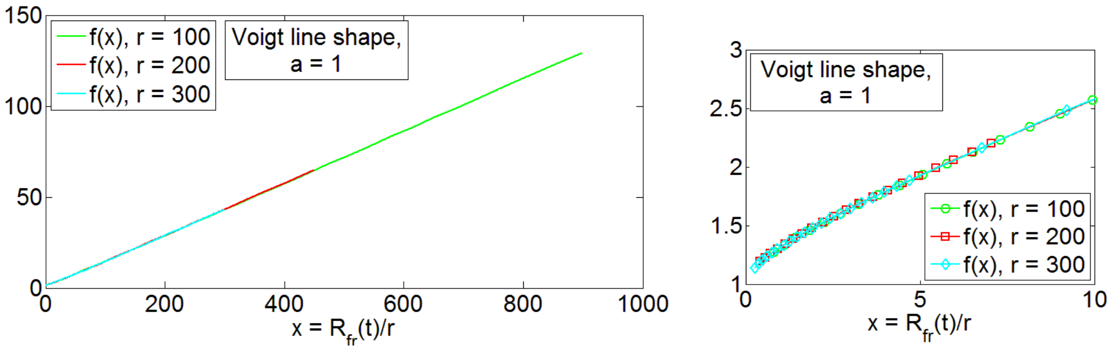
$$F_{auto}(\mathbf{r},t;\mathbf{r}_0,t_0) = (t-t_0)\cdot G\left(|\mathbf{r}-\mathbf{r}_0|\cdot f\left(\frac{r_{\mathrm{fr}}(t-t_0)}{|\mathbf{r}-\mathbf{r}_0|}\right)\right). \tag{5}$$

The results of the reconstruction of the function $f$ from comparison of the function (5) with numerical calculation of the Green function [5] for the Doppler, Lorentz, Voigt, and Holtsmark line shapes are shown in figure 2.

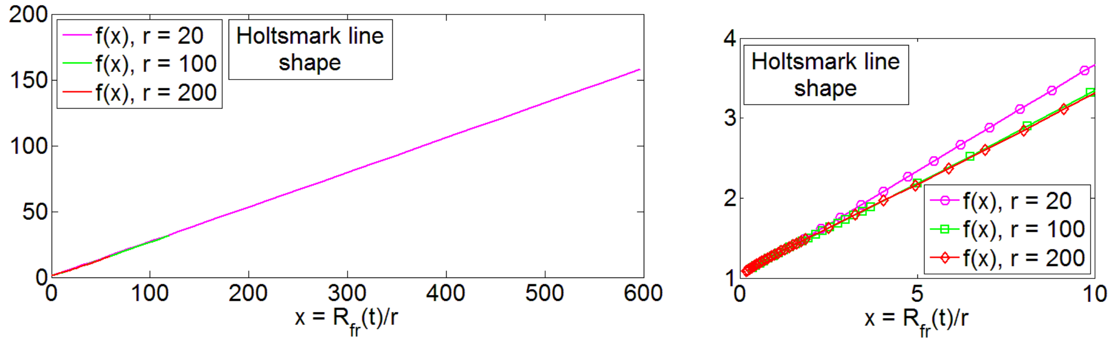



(a)

(b)




(c)




(d)

Figure 2. Reconstruction of the argument of automodel function (5) from its comparison with the exact Green function [5] for various spectral line shapes: (a) Lorentz; (b) Doppler; (c) Voigt for $a = \sqrt{\ln 2}\,\Delta\omega_{Lorentz} / \Delta\omega_{Doppler} = 1$, (d) Holtsmark.

**3. Conclusions.** The Green function [5] of the non-stationary Biberman-Holstein equation in an infinite homogeneous medium is shown to possess an automodel form for various spectral line shapes in a broad range of propagation time and distance from the source. The simplicity of the auto-model Green function suggests the possibility to construct a universal algorithm for numerical simulation of the B-H solution in the case of non-stationary and stationary RT in a finite medium.

## References.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