Mode structure of a short laser pulse propagating through a metal capillary

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Introduction

Acceleration of particles in plasmas is now of great interest thanks to plasma capability of supporting electric fields orders of magnitude stronger than those in conventional radio-frequency structures [1, 2]. One of studied schemes is laser driven plasma wakefield acceleration (LWFA) in narrow capillaries. In this scheme, a short laser pulse propagates along the capillary filled by the plasma and drives a high-amplitude Langmuir wave with the phase velocity approximately equal to the light velocity c. The capillary prevents diffraction of the laser pulse and extends the acceleration length either directly by reflecting the pulse from capillary walls [3–7], or indirectly through a specific plasma profile inside [8].

The theory of wave propagation in metallic or ionized capillaries at conditions of interest for the wakefield acceleration is incomplete yet. The classical waveguide theory [9] is not fully applicable to these conditions, as is shown in Ref. [10]. Corrected attenuation rates are obtained either with simplifying assumptions [10–12], or numerically with taking into account additional effects [12, 13]. At the same time, there are experimental evidences that attenuation of short, high-contrast laser pulses in metallic [14] or ionized [15] capillaries is low enough to study narrow capillaries as an option for LWFA.

In this paper, we consider the structure of capillary modes in cylindrical capillaries and calculate attenuation rates. We present exact solutions and analyze precision of often used approximate solutions. The depth of study comes at the sacrifice of generality: we focus only at laser and capillary parameters of interest for wakefield acceleration.

Circular capillaries

Consider a circular waveguide of the radius a with the relative dielectric permittivity $\varepsilon = 1$ inside and $\varepsilon = \varepsilon_w$ for $r \geq a$. Solving Maxwell equations in the cylindrical geometry for perturbations of the form $f(r)e^{ikz-i\omega t+im\phi}$ and taking into account continuity of E_ϕ, E_z, B_ϕ, and B_z at $r = a$ yields [16]

$$\left(\frac{J_m'}{J_m}+\frac{\varkappa_1}{\varkappa_2}\frac{K_m'}{K_m}\right)\left(\frac{1}{\varepsilon_w}\frac{J_m'}{J_m}+\frac{\varkappa_1}{\varkappa_2}\frac{K_m'}{K_m}\right)=\frac{1}{\varepsilon_w}\left(\frac{mkc}{\omega\varkappa_1 a}\right)^2\left(1+\frac{\varkappa_1^2}{\varkappa_2^2}\right)^2, \qquad (1)$$

where $\varkappa_j^2 = (-1)^j(-\varepsilon\omega^2/c^2 + k^2)$, $j = \{1,2\}$, $J_m(\varkappa_1 a)$, $K_m(\varkappa_2 a)$, $J'_m(\varkappa_1 a)$, and $K'_m(\varkappa_2 a)$ are Bessel function of the first kind, modified Bessel function, and their derivatives, respectively.

For solving equation (1) we need to specify ε_w. Both metals and quickly ionized solid walls are usually characterized by the Drude formula [17]:

$$\varepsilon_w(\omega) = 1 + i\frac{\omega_p^2\tau}{\omega(1-i\omega\tau)}, \tag{2}$$

where $\omega_p^2 = 4\pi n_e e^2/m = 4\pi\sigma_0/\tau$ is the plasma frequency of conduction electrons, n_e is their density, e and m are electron charge and mass, σ_0 is the conductivity, and τ is the electron collision frequency in the medium. The formula (2) correctly describes reflection of short powerful laser pulses from various materials which behave as a "universal plasma mirror" at high intensities [18].

To be specific, consider solutions of Eq.(1) in the parameter area of discussed experiments on LWFA [7, 14]. In particular, take the laser wavelength $\lambda = 850\,\text{nm}$ and the copper capillary of the radius $a \sim 15\,\mu\text{m}$. The electric field of the incident laser pulse has the same direction all over the transverse cross-section, which corresponds to azimuthal modes with $|m| = 1$ in cylindrical coordinates. The excited capillary modes must have the same azimuthal dependence, so we give most attention to $|m| = 1$ modes.

Since $a \gg \lambda$, low-order waveguide modes are almost plane waves and similarly have $k \approx \omega/c$. The copper at high frequencies is characterized by $\sigma_0 \approx 1.6\times10^{17}\,\text{s}^{-1}$ and $\tau \approx 1.3\times10^{-14}\,\text{s}$. [10] For these values, $\varepsilon_w \approx -30 + 1.1i$, $|\varepsilon_w| \gg 1$, $\varkappa_2 \gg \omega/c \gg 1/a$, which means the perturbation penetrates the walls a short distance. At this case, the Leontovich boundary conditions [19] are commonly used:

$$E_\phi = \zeta B_z, \qquad E_z = -\zeta B_\phi, \tag{3}$$

where $\zeta = 1/\sqrt{\varepsilon_w}$ is the surface impedance. Using the conditions (3) is equivalent to the large-argument approximation for the modified Bessel functions, $K'_m/K_m \approx -1$, approximating $\varkappa_2^2$ by $-\varepsilon\omega^2/c^2$, and neglecting the ratio $\varkappa_1^2/\varkappa_2^2$ in the right-hand side of Eq. (1).

At high conductivity of the walls, there could be two small parameters in the problem: the impedance $|\zeta|$ and the ratio $\varkappa_1 c/\omega$. Depending on the ratio of the two, solutions of Eq. (3) take qualitatively different forms. If $|\zeta| \ll \varkappa_1 c/\omega$ (very high conductivity), the problem reduces to the classical result of waveguide theory [19]: there are two groups of modes, *TM* and *TE* modes. The wave amplitude attenuates as $e^{-\alpha z}$ with

$$\alpha = \frac{\omega\,\text{Re}(\zeta)}{kac}, \qquad \alpha = \frac{c\varkappa_1^2\,\text{Re}(\zeta)}{\omega ka}\left(1 + \frac{m^2\omega^2}{c^2\varkappa_1^2(a^2\varkappa_1^2 - m^2)}\right) \tag{4}$$

for *TM* and *TE* modes, respectively.

In the case $|\zeta| \gg \varkappa_1 c/\omega$, the solutions for $m \neq 0$ are circularly polarized waves [10]:

$$J_{m\pm 1}(\varkappa_1 a) = 0, \qquad \vec{B} = \pm i\vec{E}, \qquad E_r = \pm iE_\phi, \qquad \alpha = \frac{\varkappa_1^2 \mathrm{Re}(\zeta)}{2k^2 a|\zeta|^2}. \tag{5}$$

If $m > 0$, the solutions corresponding to the upper and lower signs in (5) are named L and R modes, respectively [10]. Only R modes can be excited by a Gaussian pulse. As $\delta \equiv \frac{\varkappa_1 c}{|\zeta|\omega}$ decreases, TM_{mn} modes are continuously transformed into R_{mn} modes, where subscripts m and n denote azimuthal and radial mode numbers [10].

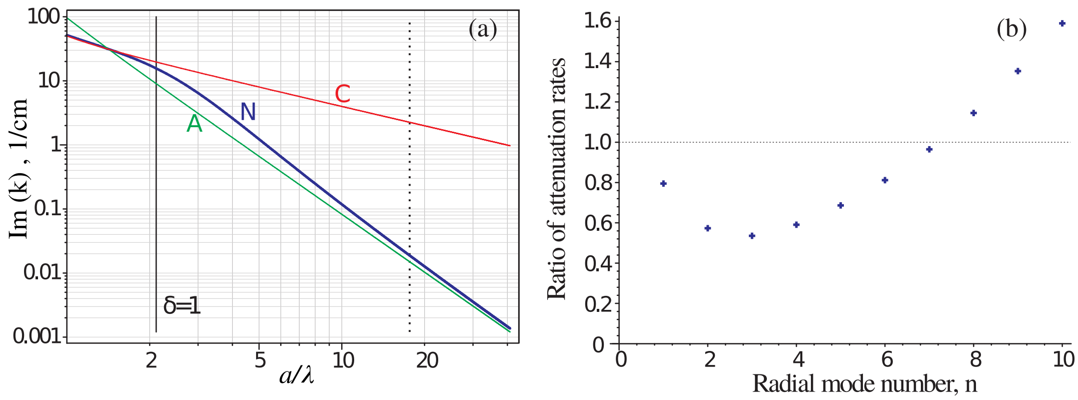

Figure 1: *(a) Attenuation rate for modes R_{11} or TM_{11} calculated from the classical formula (4) ("C"), approximate expression (5) ("A"), and numerically solved Eq. (1) ("N"). The thin vertical line mark the boundary between approximations ($\delta = 1$), the dotted vertical line shows the considered parameter set. (b) Ratio of attenuation rate obtained approximately to that obtained numerically for various R modes and the baseline parameter set.*

For the considered parameter set, the condition $|\zeta| \gg \varkappa_1 c/\omega$ is fulfilled, but with no large margin even for the lowest mode (R_{11}) with $\varkappa_1 a \approx 2.40483$, $\zeta \approx 0.0032 - 0.18i$, $\delta \sim 0.12$. This raises the question of how precise the approximate attenuation rate (5) is. To answer, we compare the exact numerical solution of Eq. (1) and its approximations for various ratios a/λ [Fig. 1(a)]. The approximate expression always underestimates attenuation. Although the graphs are close, this is the logarithmic scale, and the difference is quite noticeable. The approximate formula (5) is close to the exact solution only for

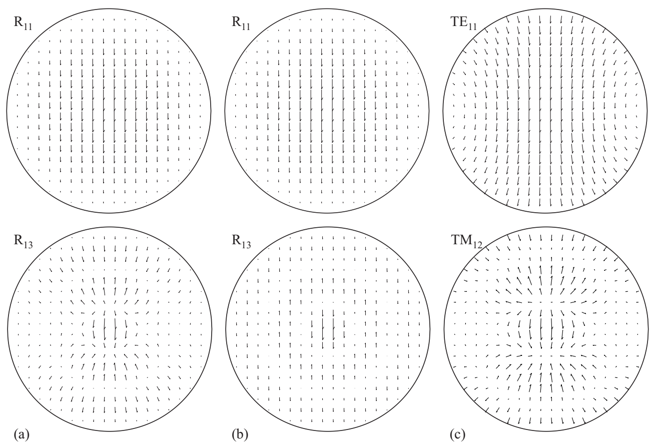

Figure 2: *Transverse electric fields for different modes calculated from exact equations (1) (a), approximation (5) (b), and classical approach (4) (c).*

the lowest order radial mode (the error about 20%). For higher order modes, the error is large [Fig. 1(b)]. Curiously, the approximate expression is correct for R modes with $n \sim 7$, for which $\delta \sim 1$, and inequality $|\zeta| \gg \varkappa_1 c/\omega$ is not valid.

The difference between exact and approximate solutions is also visible in the mode structure, but for $n > 1$ only (Fig. 2). The lowest order modes R_{11} and TE_{11}, which contain most of the incident energy in corresponding limiting cases, look similarly. The main difference is that in the limit $\delta \ll 1$ there is no electric field on the walls. For higher modes (second row in Fig. 2), the exact solution contains features of both approximations: the field vectors are noncollinear, as in TM modes, and the field vanishes at the walls, as (5) implies.

Conclusion

We have demonstrated the method of calculating the exact mode structure. The exact numerical solution differs from approximate and standard solutions. The approximate solution overestimates the length of laser pulse propagation in the parameter area of interest for LWFA. Nevertheless, the damping rate is still order of magnitude smaller than obtained from standard formulae of the classical waveguide theory. Properties of lowest order eigenmodes is acceptable for LWFA.

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