Towards a universal collision-free sheath solution with warm ions

N. Jelić[1,2], L. Kos[1], S. Kuhn[2]

[1] *Faculty of Mechanical Engineering, University of Ljubljana, SI-1000 Ljubljana, Slovenia,*

[2] *Institute for Theoretical Physics, University of Innsbruck, A-6020 Innsbruck, Austria*

By employing a numerical method for solving the collision-free plane discharge with Maxwellian electrons and a neutral Maxwellian ion source, and manipulating the results obtained, it has recently been found that the electric field E of the sheath region can be expressed in terms of the sheath potential φ via the equation $\varepsilon^2 E^2/2 + n_e T_e/2 = A\varphi$ [1] (with n_e, T_e the electron density and temperature, $A \approx \mathrm{e}^{-\varphi_s}/4$, and φ_s the plasma-edge potential [1, 2]). Keeping in mind that up to now no appreciable attempts towards formulating a consistent sheath theory with warm ions have been reported, this finding represents a milestone towards developing a possible universal or at least a conceivable parametric sheath solution. In a collision-free sheath theory, this task requires formulation of a physically plausible analytic ion velocity distribution function (VDF) at the plasma-edge boundary, such as the familiar Harrison and Thompson (HT) VDF [3] (obtained with the assumption of an absolutely cold ion source, $T_n = 0$) and the "analytic Bissel and Johnson (BJ)" VDF with warm ions (see more in [1, 2] and references therein), obtained here with a Maxwellian ion source of arbitrarily high temperature ($T_n > 0.5T_e$), in the respective forms

$$f_i(v) = \frac{2\sqrt{2}}{\pi}\left[\frac{1}{2\sqrt{\varphi - \frac{v^2}{2}}} - F_D\left(\sqrt{\varphi - \frac{v^2}{2}}\right)\right], \quad f_i(v) = \frac{L/L_i(\beta)}{\sqrt{2\pi T_n}}\int_0^{\varphi_L} \frac{d\varphi' \mathrm{e}^{-\beta\varphi'}}{E(\varphi')}\frac{\mathrm{e}^{-(\varphi'-\varphi+\frac{v^2}{2})/T_n}}{\sqrt{-\varphi+\varphi'+\frac{v^2}{2}}}.$$

Here, $F_D(z)$ denotes the Dawson function and $\frac{1}{E(\varphi)} = \frac{2T_n+1}{\sqrt{2\pi T_n}}\mathrm{e}^{(1+\frac{1}{2T_n})\frac{\varphi_s}{2}}\mathrm{e}^{(\beta-1)\varphi}\sqrt{\frac{\varphi_s-\varphi}{\varphi}}$ is our analytic electric field, taking roles in the first and second expressions, respectively, while other quantities are formulated and normalized in [1, 2]. The main parameter of the BJ problem is the (normalized) source temperature T_n (with the HT discharge, $T_n = 0$, being a special limit). Both the HT and BJ VDFs are valid at any arbitrary point φ in the quasi-neutral plasma region including the plasma edge φ_S, while their values and their corresponding moments [e.g., densities $n_{i,e}/n_0 \leftrightarrow n_{i,e}(\varphi)$, directional velocities $u_i/\sqrt{kT_e/m_i} \leftrightarrow u_i(\varphi)$, and temperatures $T_{i,e}/T_{e0} \leftrightarrow T_{i,e}(\varphi)$] in the sheath region are calculated from the above VDFs shifted towards the wall according to $v^2/2+\varphi$. In contrast to the referred-to numerical approach to the so-called "sheath-wall asymptote" concept/hypothesis [1], we here employ the present semi-analytic approach/solutions, being capable of resolving the apparently hidden effects of slow ions present in the ion VDFs on the charge-density profiles.

References

[1] L. Kos, N. Jelić, D. D. Tskhakaya, and S. Kuhn, Phys. Plasmas **25**, 032804 (2018).

[2] L. Kos, N. Jelić, T. Gyergyek, S. Kuhn, and D. D. Tskhakaya, AIP Advances **8**, 105311 (2018).

[3] E. R. Harrison and W. B. Thompson, Proc. Phys. Soc. **74**, 145 (1959).