Large amplitude quasi-periodic structures mediated via coherent nonlinear oscillations in 3D Magneto-Hydro-Dynamics

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We report a numerical observation of 'recurrence' phenomenon in a three dimensional magneto-hydrodynamic (MHD) plasma for certain classes of initial flow profiles. Our simulations discover such a behaviour for an initial Taylor-Green (TG) type of flow whereas under identical conditions an initial Arnold-Beltrami-Childress (ABC) flow fails to show recurrent behaviour.

1 Introduction

Understanding plasma turbulence is key to control the disruption of plasma in experimental devices consequently improving the confinement of plasma. Predicting extreme events occuring in astrophysical objects and stellar matter are of fundamental interest. Also one of the best suited models to explain various large or intermediate scale events in plasma is magnetohydrodynamics (MHD). The magnetic field lines coupled with the plasma flow offer completely new dynamics and energy transfer between modes.

A MPI-parallel three dimensional pseudo-spectral "direct numerical simulation" (DNS) code has been developed in-house that governs the dynamics of plasma within the framework of single fluid MHD model. In collaboration with NVIDIA, the code has been multi-GPU parallelised for NVLink based GPU-nodes. [Rupak Mukherjee *et. al.* Proceedings of 25th International Conference on High Performance Computing Workshops (HiPCW), 2018; arXiv:1810.12707]

For three dimensional chaotic flows at Alfven resonance, we find that for two different initial conditions, one flow reconstructs the initial fluid and magnetic flow profile and the other does not. We call the phenomena as "Recurrence" and argue the explanation of the event checking boundedness of Rayleigh quotient measuring the effective number of active degrees of freedom in a high dimensional system. [Rupak Mukherjee, R Ganesh, Abhijit

Sen; Physics of Plasmas 26, 022101 (2019) {Editor's Pick}; arXiv:1811.00754]

In principle, all periodic or quasiperiodic motion can be considered as examples of "recurrent dynamics". Such behavior is commonly found in many nonlinear driven dissipative systems. Sawteeth, ELMs and many similar phenomena in tokamaks, are prime examples of such phenomena that have been well studied numerically in this context. Periodicity results if essentially two degrees of freedom are mainly "active" whilst quasi periodicity involves a few incommensurate frequencies. Truly recurrent motions however occur when the recurrence time is long compared with typical frequencies of the system. It is such behavior found in nonlinear continuum systems that is at the heart of past studies of recurrence and is also the focus of our present investigation.

2 Governing Equations

The basic equations governing the dynamics of the single fluid MHD plasma that are time evolved in the code are:

$$\frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot (\rho \vec{u}) = 0 \tag{1}$$

$$\frac{\partial (\rho \vec{u})}{\partial t} + \vec{\nabla} \cdot \left[\rho \vec{u} \otimes \vec{u} + \left(P + \frac{B^2}{2} \right) \mathbf{I} - \vec{B} \otimes \vec{B} \right] = \mu \nabla^2 \vec{u} \tag{2}$$

$$P = C_s^2 \rho \tag{3}$$

$$\frac{\partial \vec{B}}{\partial t} + \vec{\nabla} \cdot \left(\vec{u} \otimes \vec{B} - \vec{B} \otimes \vec{u} \right) = \eta \nabla^2 \vec{B} \tag{4}$$

3 Numerical Result

3.1 Parameter details

The parameters for which we run our code G-MHD3D are given in Table 1.

N	L	dt	ρ_0	U_0	Re	Rm	M_s	M_A
64	2π	10^{-5}	1	0.1	450	450	0.1	1

Table 1: Parameter details for the results mentioned in this report. These parameters are kept identical throughout this report unless stated otherwise.

3.2 Taylor-Green flow

$$
\begin{aligned}
u_x &= A\ U_0\,[\cos(kx)\sin(ky)\cos(kz)] \\
u_y &= -A\ U_0\,[\sin(kx)\cos(ky)\cos(kz)] \\
u_z &= 0
\end{aligned} \tag{5}
$$

We choose $A = 1$ and $k = 1$ in this study.

3.3 Arnold-Beltrami-Childress flow

$$
\begin{aligned}
u_x &= U_0[A\sin(kz) + C\cos(ky)] \\
u_y &= U_0[B\sin(kx) + A\cos(kz)] \\
u_z &= U_0[C\sin(ky) + B\cos(kx)]
\end{aligned} \tag{6}
$$

$A = B = C = 1$ and $k = 1$ are chosen for this study.

4 Summary and Conclusion

We report the observation of recurrence of velocity and magnetic field isosurfaces for a three dimensional nearly ideal magnetohydrodynamic plasma. The initial flow profile plays a crucial role in the recurrence phenomenon.

The generation of the flows considered in this study can be reproduced in laboratory experiments. This opens up the possibility of observation of recurrence in experimental plasma devices also.

The periodic reconstruction of structures indicates a low dimensional behaviour of a high degree of freedom system thereby indicating a possibility of short time prediction of plasma states in any plasma experiment. A detailed numerical and analytical study is already available [Rupak Mukherjee, R Ganesh, Abhijit Sen; Physics of Plasmas 26, 042121 (2019); arXiv:1811.00744]. This may help in increment of plasma confinement study and apriori identification of disruptive structures in experimental plasma devices.

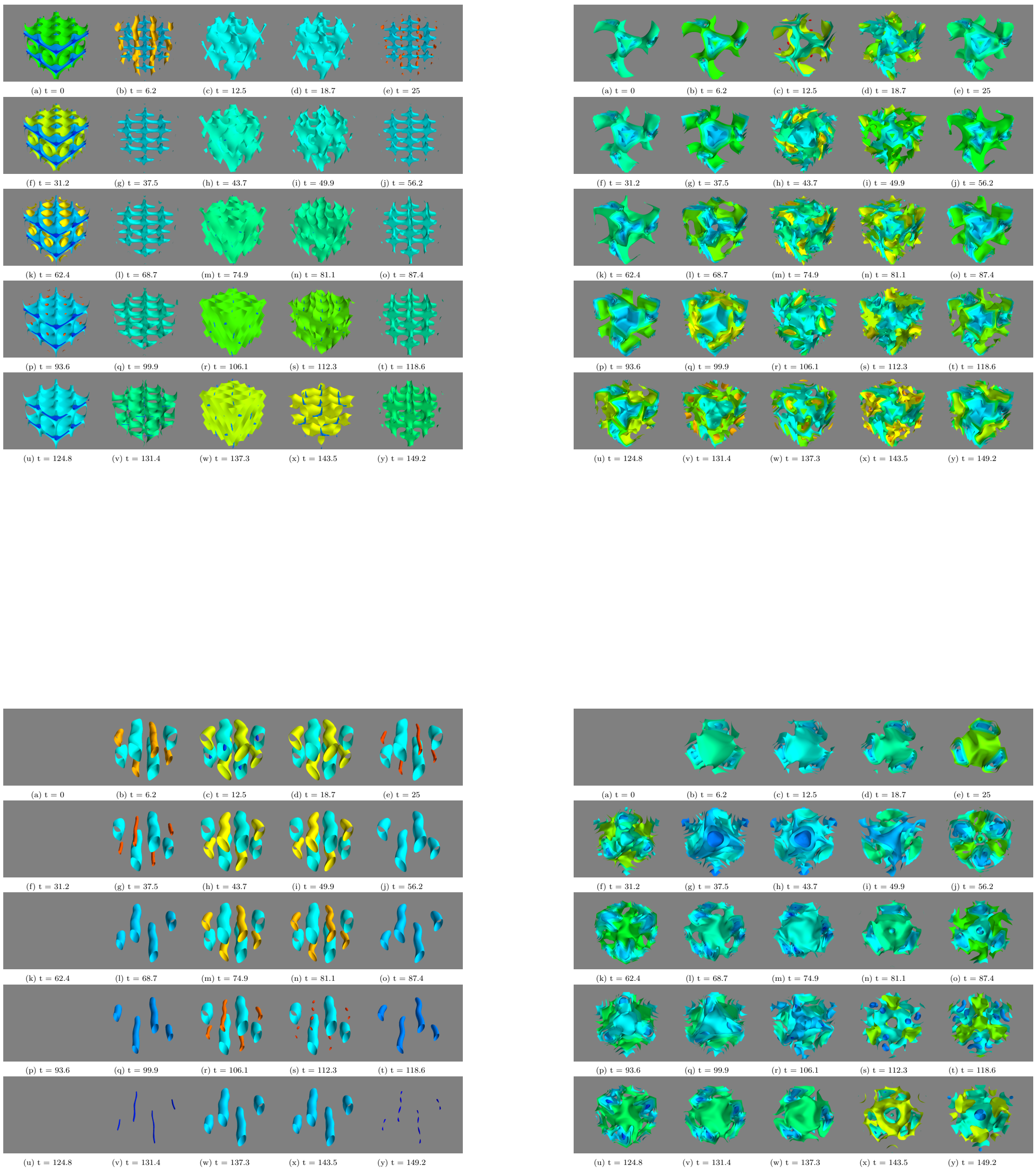

Figure 1: (Left) TG flow showing periodic recurrence with time. (Right) ABC flow fails to show recurrence under the identical condition.