Piece-wise power law or interpolated curve, does it matter for optically thin radiative cooling?

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Abstract. Optically thin radiative cooling can drive condensation phenomena. In the literature analytic piece-wise power laws as well as interpolatable tables are used. We investigate whether this makes a difference by setting up 2D magnetohydrodynamics (MHD) simulations of condensation formation by thermal instability. The growth and damping rates derived from power laws versus tables are different. However thermal instability is initiated in both cases. The filaments fragment whenever we employ cooling curves that are not artificially vanishing at low temperatures. The morphology of the fragmented filaments in the nonlinear regime is only slightly affected.

Introduction

Optical thin radiative cooling is an important physical process because of its role in the formation of condensations, which are observed in many astrophysical environments. An example in solar physics are prominences [1]. Instead of solving the full radiative transfer equations, precomputed cooling curves can be used in numerical simulations. They typically are implemented in one of two ways. First there are interpolatable tables, such as the SPEX_DM curve [2], which are interpolated to a high temperature-resolution from a given amount of data points. Second there are analytical piece-wise power laws, such as the Rosner curve [3]. In this work we study how the implementation of a cooling curve affects the formation of condensations. As mechanism to form condensations we chose thermal instability, first described by Field [4] and recently revisited by Claes et al. [5, 6]. We will use a setup where two slow MHD modes perturb a uniform medium typical for the solar corona. The slow modes will damp and the thermal mode will lead to a dense condensation. The condensation will have a filamentary shape and in the nonlinear stage fragment due to ram pressure differences. This setup has also been used by Hermans et al. [7], where the effect of

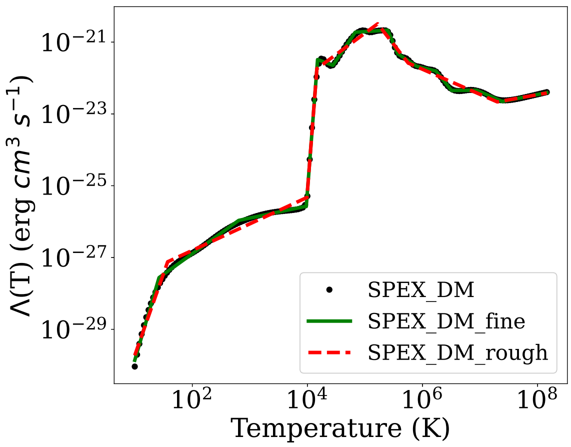

Figure 1: *The original data points of the SPEX_DM cooling curve and the two piece-wise power law fits.*

cooling curves on condensation formation was investigated. Changing cooling curve changes the time scale at which this process happens. It was also shown that the low temperature treatment of the cooling curve influences the fragmentation of the condensation.

Cooling curves and theoretical growth rates

We compare two piece-wise power laws fits with two interpolated tables, with 1000 and the default 4000 points. We fitted the original, interpolatable SPEX_DM cooling curve [2], consisting out of 180 points, using a segmented regression method [8]. By choosing the number of segments we created a 'fine' and a 'rough' piece-wise power law fit, consisting out of 14 and 7 segments respectively[1]. The data points of the original SPEX_DM curve together with the two piece-wise power laws are shown in Fig. 1. The 'rough' fit still follows the general behaviour of the curve but is much less refined than the 'fine' fit. Differences in the cooling rate, but most importantly in the slopes of the segments, i.e. the derivative of the cooling rate, will influence the growth and damping rates of the condensation and MHD modes. To quantify this difference we solved the dispersion relation as derived by Claes et al. [5] for a uniform unbounded medium with physical parameters discussed in the next section. In Fig. 2 the complex eigenfrequency plane is shown. It can be seen that there is almost no difference between using 1000 and 4000 points because the cooling curves are nearly the same. For the piece-wise cooling curves it does differ. The slow and fast MHD modes are more damped. The thermal mode is less unstable. The percentual relative difference with respect to the 4000 points curve is about 10% for the slow and fast MHD modes, but 30% for the thermal mode for the 'rough' piece-wise power law. For the 'fine' piece-wise power law it is approximately 20% for all three modes.

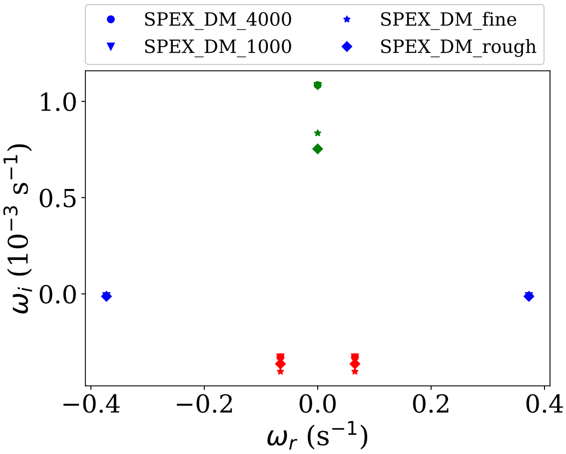

Figure 2: *The complex eigenfrequency plane. The thermal, slow and fast mode are denoted by the green, red and blue symbols, respectively.*

Numerical setup

For the four cooling curves we simulate the in-situ condensation forming process by thermal instability to see how this fundamental process is affected. We follow the methodology of Claes et al. [5, 6] and Hermans et al. [7]. We make use of the MPI-parallelized Adaptive Mesh Refinement Versatile Advection Code (MPI-AMRVAC)[2] [9] to numerically solve the MHD equations.

[1]The analytic formulae can be found at https://erc-prominent.github.io/team/jorishermans/
[2]Open source at http://amrvac.org

Our initial conditions and numerical methods are the same as used for the benchmark simulation of Hermans et al. [7], with the exception that we use 4 instead of 6 levels of AMR in this work. The setup is a 2D box with periodic boundary conditions and solar coronal conditions, i.e. a temperature of 10^6 K and a density of 2.34 10^{-15} g cm^{-3}. The strength of the background magnetic field is set to 10 G and the angle between the x axis and and the magnetic field is $\pi/4$. Upon this uniform equilibrium we superimpose two adiabatic slow MHD waves. These initially propagate along the x and y directions, but while they interfere, they damp out due to the radiative losses. Eventually, the unstable thermal mode kicks in and condensation happens.

Results

For the four cooling curves the plasma is unstable with respect to both the isochoric and isobaric instability criterion. Hence, as expected, in all cases a cool and dense condensation with a filamentary shape is formed. It also fragments in all cases and this leads to a very small time step restriction, as described by Hermans et al. [7]. The density view at the end of the evolution for the four cooling curves is shown in Fig. 3. There is little difference in the fragmentation pattern between the simulations using the interpolated cooling curves with 4000 and 1000 points. Those small differences arise due to the erratic nature of the small numerical seeds that drive the fragmentation. The bottom two panels show the piece-wise power laws. The morphology of their filaments is comparable to each other, but slightly different to the shapes of the filaments of the interpolated cooling curves. However it is important to note that for both the piece-wise power laws and the interpolated tables the filament fragments. This is in contrast to simulations using cooling curves which vanish at low temperatures, such as the Rosner curve [3], as discussed in Hermans et al. [7].

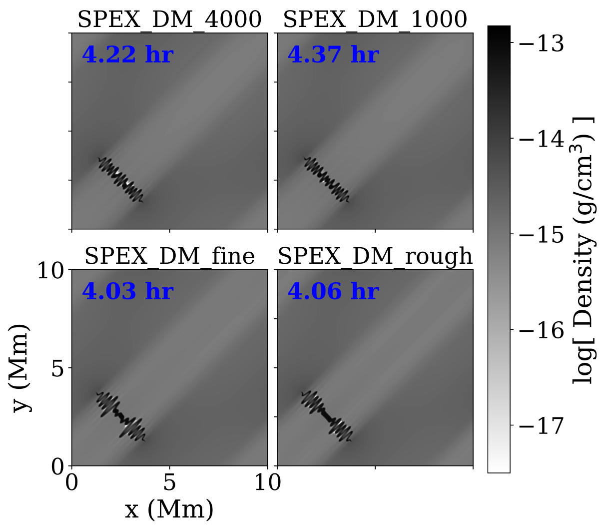

Figure 3: *The density view at the end of the evolution for the four cooling curves.*

In Fig. 4 the evolution of maximum density and minimum temperature for the four simulations is shown. It is clear that in all four cases the quantities evolve similarly, if not in exactly the same way. The only difference to remark is the time scale on which the processes happen. As mentioned previously, the growth and damping rates of the thermal and slow MHD modes are different for interpolated and piece-wise power laws. Hence it is expected that the onset of thermal instability happens at a different moment in time. The difference between simulations using the interpolated curves with 1000 and 4000 points is most likely because the cooling curve with 1000 points has a lower temperature-resolution. This difference propagates and becomes large after a lot of time steps (± 412000).

Conclusions

The growth and damping rates of MHD modes are affected by the choice of implementation because the cooling rate and its derivative at the equilibrium temperature can vary, especially the latter. However thermal instability is robust, easy to trigger, in an ideal setup with only radiative cooling and background heating. Various additional ingredients, such as thermal conduction, can stabilise thermal instability (see e.g. Claes et al [6]), but these are here omitted to isolate the runaway radiative loss effects. For both piece-wise power laws and interpolated curves the filament fragments. They fragment in a similar way. Hence the numerical implementation does not significantly affect the fragmentation process. However this confirms the importance of the low temperature treatment of cooling curves [7].

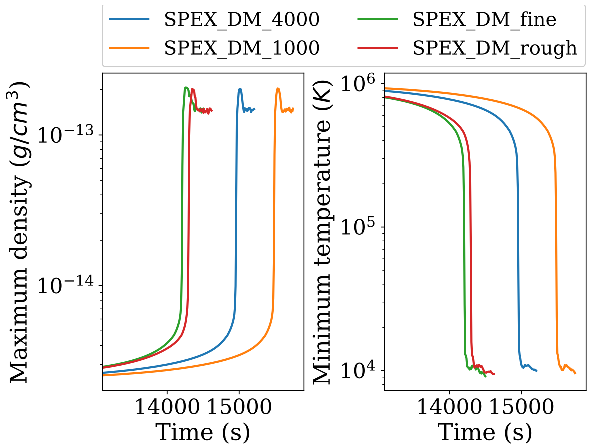

Figure 4: *The left and right panels show the evolution of, respectively, the maximum density and minimum temperature for the four simulations.*

Acknowledgements. The visualisations were made using the open source software Python (python.org) and yt (yt-project.org). The authors are supported by the ERC Advanced Grant PROMINENT and a joint FWO-NSFC grant G0E9619N. This project has received funding from the European Research Council (ERC) under the European Union's Horizon 2020 research and innovation programme (grant agreement No. 833251 PROMINENT ERC-ADG 2018). This research is supported by Internal funds KU Leuven, project C14/19/089 TRACESpace.

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