Runaway positrons in tokamak plasmas

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Runaway positrons can be produced in the presence of runaway electron avalanches in magnetized plasmas. Almost all the positrons generated by avalanching runaways will run away and are expected to have lifetimes of several seconds. For an avalanching positron distribution typical of tokamak plasmas the maximum of the synchrotron radiation spectrum should be around a micron. The radiated power is sensitive to many plasma parameters, specially the number of impurities, temperature and density. Apart from its intrinsic interest, detection of radiation from positrons could be a diagnostic tool to understand the properties of the annihilation medium.

Introduction Relativistic electron populations originating from runaway electron avalanches have been frequently observed in various plasmas, e.g. large tokamak disruptions. In post-disruption plasmas in large tokamaks, the energy of the runaway electrons is in the $10-20\,\mathrm{MeV}$ range. Therefore in these plasmas the typical runaway energy is well above the threshold for pair-production and positrons should therefore be present in large quantities [1]. The positrons generated by runaway electron avalanches are highly relativistic already at birth, and in addition they experience acceleration by the electric field. In the present work we calculate the distribution of positrons and the synchrotron radiation emitted by them. The production rate is calculated by using a pair-production cross-section valid for arbitrary energies and a runaway electron distribution typical for avalanching. To obtain the positron velocity distribution, the Fokker-Planck equation including the positron production and annihilation rates and slowing-down terms is solved. The result is used to calculate the fraction of runaway positrons and the parametric dependences of their synchrotron radiation spectrum.

The cross-section for the production of electron-positron pairs by electrons in the field of a nucleus is $\sigma_{\rm tot} = aZ^2 \ln^3[(\gamma_e + x_0)/(3+x_0)]$, where $a = 5.22\,\mu b$ ($1b = 10^{-28}\,\mathrm{m}^2$), $x_0 = 3.6$, Z is the charge of the stationary particle, and γ_e is the Lorentz factor of a fast electron [2]. The positron production rate $dn^+_{\rm prod}/dt \equiv S_p$ is given by $S_p = n_i \int_{p>3} f_e^{\rm RE} \sigma_{\rm tot} v_e d^3 p_e$, where n_i is the number density of the ions. Here $f_e^{\rm RE}$ is the runaway electron distribution function, $p_e = \gamma_e v_e / c$ is the normalized relativistic momentum, c is the speed of light. If the normalized parallel electric field $E = e|E_\parallel|\tau/m_e c \gg 1$, the runaway tail has the character of a beam, so the parallel momentum is much larger than the perpendicular, $p_{e\perp} \ll p_{e\parallel} \simeq p_e$. Here, $\tau = 1/4\pi r_e^2 n_e c \ln\Lambda$ is the collision time for relativistic positrons and electrons and $\ln\Lambda$ is the Coulomb logarithm. If $E \gg 1$ most of the runaway electrons are produced by avalanching, in which case the runaway electron density n_r increases according to $dn_r/dt \simeq n_r(E-1)/c_z\tau\ln\Lambda$, where $c_z = \sqrt{3(Z_{\text{eff}}+5)/\pi}$ and Z_{eff} is the effective ion charge. Then the distribution function of relativistic runaway electrons is

$$f_e^{\text{RE}}(p_{e\parallel}, p_{e\perp}) = \frac{n_r \hat{E}}{2\pi c_z p_{e\parallel} \ln\Lambda} \exp\left(-\frac{p_{e\parallel}}{c_z \ln\Lambda} - \frac{\hat{E} p_{e\perp}^2}{2p_{e\parallel}}\right), \tag{1}$$

where $\hat{E} = (E-1)/(1+Z_{\text{eff}})$ [3]. The runaway electron density n_r depends on the runaway current I_r and the major radius of the torus R. As an example, in a post-disruption plasma with and $I_r = 1$ MA and $R = 3$ m we have $N_r = 2\pi R I_r/ec = 4\times10^{17}$. Assuming that the runaways are concentrated in a beam of total volume $1\ \text{m}^3$, the runaway density is $n_r = 4\times10^{17}\ \text{m}^{-3}$. Taking only account collisions between runaway electrons and hydrogenic ions with density $n_i = 5\times10^{19}\ \text{m}^{-3}$, we find that the production rate is $S_p = (n_r n_i c/c_z \ln\Lambda)\int_3^\infty e^{-p/c_z\ln\Lambda}\sigma_{tot}dp \simeq 1.5\times10^{13}\ \text{s}^{-1}\text{m}^{-3}$ (for $\ln\Lambda = 10$ and $Z_{\text{eff}} = 1.6$). Also, collisions with thermal electrons and impurities give contribution to the number of positrons created. The number of positrons created in collisions with electrons is about the same order of magnitude as that from collisions with hydrogenic ions (although the threshold momentum is higher, most of the runaway electrons typically do exceed that as well). S_p should therefore be multiplied by $M_p \equiv 1 + n_e/n_i + \sum_z n_z Z^2/n_i$, where the summation is over all impurity species (regardless if they are fully ionized or not). Due to the substantial amount of high-Z impurities present in the post-disruptive plasmas, this multiplicative factor can be several orders of magnitude. If we assume that during the tokamak disruption, at least 1 g carbon is released from the wall and it is distributed uniformly in a volume of about $80\ \text{m}^3$, that would correspond to a multiplicative factor of $M_p \simeq 450$. Note, that M_p can be large even if Z_{eff} is order unity, because the expression for M_p contains the full nuclear charge.

For the energies of interest ($\gamma_+ \lesssim 50$) collisional slowing down dominates, and the positron distribution function can be calculated from the kinetic equation $\partial f_+/\partial t = s_p(p_+) - n_e v_+ \sigma_{an} f_+ + \tau^{-1} p_+^{-2}\partial/\partial p_+ \left[(1+p_+^2) f_+\right]$, where the first term on the right is the production rate, the second is annihilation and the last is the slowing down. If the probability distribution of positrons with momentum p_+ generated from electrons of momentum p_e, $\mathcal{F}(p_e, p_+)$, is known the positron velocity distribution can be calculated by using $s_p \equiv df_+/dt = n_i \int f_e^{RE} \sigma_{tot} v_e \mathcal{F}(p_e, p_+) dp_e$ as the source term in the kinetic equation. The mean energy of the positrons generated by an incoming electron with momentum E_e is $\langle E_+ \rangle = E_e/3 - 0.0565 E_e \ln(E_e/3m_e c^2)$ [2]. A simple estimate for the positron source can be found by solving the integral in s_p assuming that $\mathcal{F}(p_e, p_+)$ is a delta-function at a point when the expression for the mean energy $\langle E_+ \rangle$ given above is satisfied. This means that s_p can be approximated by $s_p^\delta = \int f_e^{RE} \sigma_{\rm tot} v_e \delta\left(p_e - 4.42 p_+^{1.445}\right) 4\pi p_e^2 dp_e$. Note, that the production rate S_p is related to s_p through $dn_+/dt = S_p = \int (df_+/dt) d^3 p_+$. The solution of the kinetic equation is given in Ref. [4] and the result shows that the total number of positrons in the above mentioned example (for $n_e = 5 \times 10^{19}\ \mathrm{m}^{-3}$) is $n_+ = 8 \times 10^{12} M_p\ \mathrm{m}^{-3}$.

When the electric field is neglected, the distribution function is isotropic in velocity space, whereas a runaway tail is pulled out in the direction of the magnetic field if the electric field is taken into account. The positrons will run away in the opposite direction to the electrons. We can estimate the number of positrons that run away by investigating how many positrons have velocities above the critical velocity $v_c = c/\sqrt{2E}$. Since $p_c \ll p_+$, almost the whole positron population can be expected to run away $n_{run}^+ \simeq n_+$. Runaway positrons are expected to live long since the lifetime is inversely proportional to the annihilation cross section which is a strong function of positron energy. The synchrotron radiation of runaway positrons is peaked in the direction opposite from that of the runaway electrons and it may be possible to detect. For one positron, with Lorentz factor $\gamma_+ = 10$ and $v_\perp / v_\| = 0.1$, the maximum of the radiation spectrum is around 100 μm, but the velocity-integrated synchrotron spectrum for a beam-like distribution has a maximum at a lower wavelength, around 1 μm. The left figures in Fig. 1 show the synchrotron radiation spectrum and its sensitivity to the Coulomb logarithm, electron density and effective charge. As expected, the magnitude of the radiated power is larger in plasmas with a large number of impurities, high temperature and density. The right figures show the relative spectrum intensity to illustrate that not only the absolute magnitude but also the spectrum shape is dependent on the plasma parameters. The total radiated power for the parameters used here is around $0.2 M_p$ W.

Conclusions Positron radiation measurements, along with other diagnostics, could become a tool to better understand plasmas containing runaway electrons. These plasmas usually are characterized by sudden cooling and various instabilities, and are notoriously hard to diagnose. Dedicated measurements of positron radiation may therefore lead to important new insights in the processes that are particular for these plasmas.

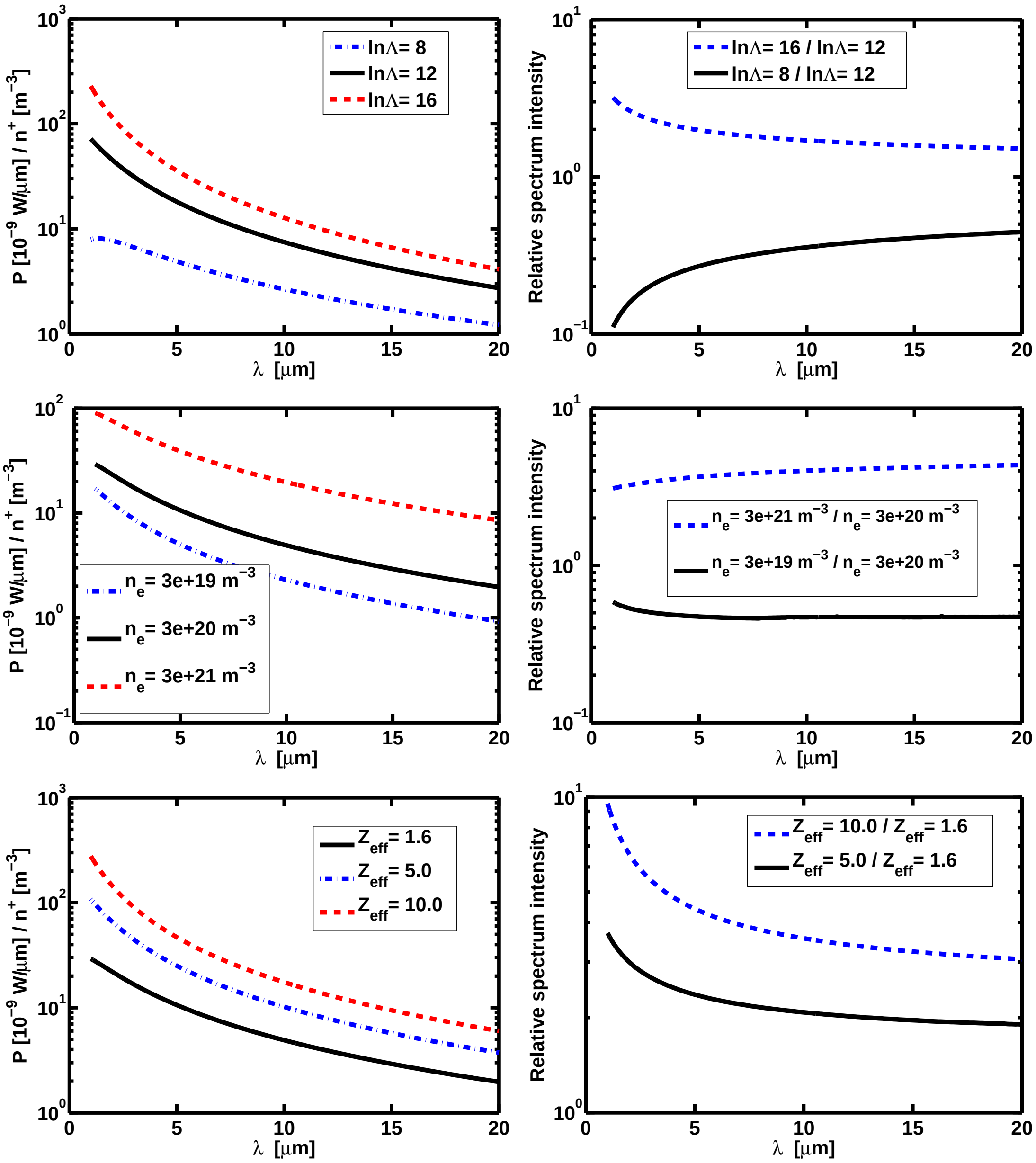

Figure 1: Synchrotron radiation spectrum normalized to the positron number density, for an avalanching distribution. Unless otherwise stated, the parameters are the following: magnetic field $B = 2.27\,\mathrm{T}$, *parallel electric field* $E_{\|} = 20\,\mathrm{V/m}$, *major radius* $R = 1.8\,\mathrm{m}$, *effective charge* $Z_{\mathrm{eff}} = 1.6$, *Coulomb logarithm* $\ln\Lambda = 10$, *and electron density* $n_e = 3\times10^{20}\,\mathrm{m}^{-3}$. *The rest of the parameters are changed according to the legend of the figures.*

References

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