

Nonlinear ECRH and ECCD Modeling in Toroidal Devices*

R. Kamendje¹, S. V. Kasilov², W. Kernbichler¹, E. Poli³, M.F. Heyn¹

¹*Institut für Theoretische Physik, Technische Universität Graz
Petersgasse 16, A-8010 Graz, Austria*

²*Institute of Plasma Physics*

NSC “KhIPT”, Ul. Akademicheskaya 1, 61108 Kharkov, Ukraine

³*Max-Planck-Institut für Plasmaphysik*

Boltzmannstrasse 2, D-85740 Garching bei München, Germany

1. Introduction

In high power ECCD scenarii using the 2nd harmonic electron cyclotron resonance for the extraordinary mode (X-mode), the applicability of the linear theory of wave absorption and of the quasilinear theory of the evolution of the distribution function is violated in a significant part of the electron velocity space at present day experiments with power levels as applied in ASDEX Upgrade, W7-AS or Tore Supra. Moreover, in reactor-scale applications of ECRH and ECCD such as, for instance, in ITER, nonlinear wave-particle effects are expected to be increasingly important owing to the scaling of the nonlinearity parameter of wave-particle interaction with the size of the device and with the value of the main magnetic field (see below). Therefore, the nonlinear nature of the wave-particle interaction resulting from a relatively strong perturbation of the wave-particle phase has to be taken into account in computations of ECCD. Further, the effects connected with this nonlinearity of wave-particle interaction are determined by local processes within the beam and therefore, are common to both stellarators and tokamaks.

A Monte Carlo method of evaluation of the electron distribution function which takes into account realistic orbits of electrons during their nonlinear cyclotron interaction with the wave beam has been proposed in Ref. [1]. The focus there was on a proper description of particle interaction with a wave beam while the geometry of the main magnetic field outside the beam was the simplest possible (slab model). In the actual work, a more realistic tokamak geometry has been implemented in the model. In addition, an expression for the parallel current density through Green's function has been used. This allows to reduce statistical errors which result from the fact that the current generated by particles with positive $v_{\parallel} > 0$ is almost compensated by the current resulting from particles with $v_{\parallel} < 0$ if the complete distribution function is taken into account in the expression for the current. The code ECNL which is a Monte Carlo kinetic equation solver based on this model, has been coupled with the beam tracing code TORBEAM [2]. The results of nonlinear modeling of ECCD in a tokamak with ASDEX Upgrade parameters with help of this combination of codes are compared below to the results of linear modeling performed with TORBEAM alone. In addition, implications for stellarators are discussed.

*This work has been carried out within the Association EURATOM-ÖAW and under Contract No. P16157-N08 with the Austrian Science Foundation. The content of the publication is the sole responsibility of its publishers and does not necessarily represent the views of the commission or its services

2. Formulation of the problem

During ECCD (ECRH), the electron distribution function is determined by resonant wave-particle interaction processes which take place in the small power deposition region as well as by the effects of particle drift motion and Coulomb collisions in the main plasma volume. Introducing Poincaré cuts at each side of the microwave beam (at positions $\varphi = \varphi_{A,B} = \text{const}$ where φ is the toroidal angle of a quasi-toroidal coordinate system) the tokamak volume is split into two regions: a narrow toroidal region where the wave particle interaction takes place, and the rest of the volume (outer region) where this interaction is negligibly small. Following Ref.[1], the kinetic equation is reduced to a set of integral relations which map the pseudo-scalar particle flux densities through the neighboring cuts, $\Gamma = |v_{\parallel}|v_{\perp}f$, where v_{\perp}, v_{\parallel} and f are the perpendicular and parallel electron velocity and the distribution function, respectively. Combined together using the periodicity of the problem, the relations which map the flux through the wave beam and through the outer region form an integral equation,

$$\Gamma^{\text{out}} = \hat{P}^H \hat{P}^O \Gamma^{\text{out}}, \quad (1)$$

where the integral operator \hat{P}^H accounts for wave-particle interaction and the operator \hat{P}^O describes the particle drift motion in the outer region taking into account Coulomb collisions. In ECNL, the integral equation (1) is solved with help of a Monte Carlo method. In order to improve the accuracy of the parallel current computation, Eq. (1) is rewritten in the form

$$\Gamma^{\text{out}} = \hat{I} \hat{P}^O \Gamma^{\text{out}} + Q^H, \quad Q^H = (\hat{P}^H - \hat{I}) \hat{P}^O \Gamma^{\text{out}} = (\hat{P}^H - \hat{I}) \Gamma^{\text{in}}, \quad (2)$$

where the unit operator \hat{I} corresponds to \hat{P}^H in the case of zero amplitude of the RF-field. For known Q^H , Eq. (2) corresponds to the stationary drift-kinetic equation with particle sources located in the outgoing regions of the cuts,

$$\left(\frac{df}{dt} \right)_D = \hat{L}_C f + Q_{\text{EM}}, \quad (3)$$

$$Q_{\text{EM}} \equiv \frac{h^\varphi}{v_{\perp}} \left[\Theta(v_{\parallel}) \delta(\varphi - \varphi_B) + \Theta(-v_{\parallel}) \delta(\varphi - \varphi_A) \right] Q^H(\vartheta, v_{\perp}, v_{\parallel}), \quad (4)$$

where $(df/dt)_D$ is a full time derivative of f due to the guiding center motion, \hat{L}_C is the Coulomb collision operator, ϑ is the poloidal angle, h^φ is the contra-variant toroidal component of the unit vector along the magnetic field, and Θ is a Heaviside step function. The solution of (3) in terms of Green's function gives the following expression for the current (compare with Ref. [3])

$$\frac{j_{\parallel}}{B} = \int_0^{2\pi} \frac{d\vartheta}{2\pi B} \int_0^{2\pi} d\varphi \int_0^{\infty} dv_{\perp} \int_{-\infty}^{\infty} dv_{\parallel} v_{\perp} g(\vartheta, v_{\perp}, v_{\parallel}) Q_{\text{EM}}(\vartheta, \varphi, v_{\perp}, v_{\parallel}), \quad (5)$$

where $g(\vartheta, v_{\perp}, v_{\parallel})$ is the generalized Spitzer-Härm function which satisfies the adjoint kinetic equation

$$\left(\frac{d(f_M g)}{dt} \right)_D + \hat{L}_C(f_M g) = -ev_{\parallel} f_M, \quad (6)$$

where f_M is a Maxwellian. The function g has been pre-computed for the long mean free path regime using the method of Ref. [4] and has been used in the form of interpolation.

3. Computational results

For the modeling, beam and plasma parameters have been chosen in the range of interest at ASDEX Upgrade. The central electron density and temperature were $6 \cdot 10^{13} \text{ cm}^{-3}$ and 2.4 keV, respectively. The safety factor was changing in the range $1 \leq q(r) \leq 4$. The wave frequency of 140 GHz was corresponding to the off-axis second cyclotron harmonic resonance in the midplane for the magnetic field value on the geometrical axis $B_0 = 2.1 \text{ T}$. The input power was $P = 0.5 \text{ MW}$, and the beam width was close to 3 cm in the absorption region. The case of low field side launch has been considered with the resonance located at the high field side for the toroidal launching angles $\phi_{inj} = -10^\circ$ and $\phi_{inj} = 0^\circ$. The results of the linear and nonlinear computation are presented in Fig. 1 and Fig. 2. In Fig. 1 it can be seen that for the beam and plasma parameter set mentioned above, nonlinear effects are not significant. This is due to the fact that power is absorbed mainly on the low field side (with respect to the position of the resonance surface for which the wave frequency is twice the cyclotron frequency) where, as Fig. 3 shows, the location of the resonance curve in velocity space is in the region of relatively large pitch-angle values for which the quasilinear approximation is well applicable, while nonlinear effects are important for relatively slow particles with pitch angles in the range between 45° and 135° . In addition, the quasilinear distortion of the electron distribution function is also not significant for this off-axis heating case. However, the onset of nonlinear effects and the consequent reduction of absorption coefficient and total ECCD current is observed if the parallel size of the beam is increased (by a factor three in the actual case). This is due to the fact that the nonlinearity parameter, $\epsilon_{NL} \sim P^{\frac{1}{4}} B_0^{\frac{1}{2}} L_b^{\frac{1}{2}} \tan \chi$, scales with the parallel beam width (L_b) with the power 1/2 (see Ref. [1]) thus increasing the range of pitch angles where nonlinear effects are important. Due to the local reduction of the absorption coefficient, the power deposition profile is shifted towards the region of low current drive efficiency (see Fig. 1), and the amount of total driven current I is reduced.

For the case of perpendicular propagation of the wave beam ($\phi_{inj} = 0^\circ$) the resonance curve shrinks to a circle whose center is located at $v_\perp = v_\parallel = 0$ in velocity space. Thus, this resonance line crosses the region of relatively small pitch-angle values where particles are subject to nonlinear interaction with the wave beam. For this case, Fig. 2 illustrates the consequent reduction of the absorption coefficient and, hence, broadening of the absorbed power density profile obtained within the nonlinear computation as compared to the results of linear modeling.

Changing B_0 to 2.157 T, the power deposition profile can be placed around the rational- q flux surface with $q = 3/2$ (see Fig. 4). A reduction of absorbed power and ECCD current densities is observed on and near the rational magnetic surface. This salient behavior is due to the fact that near rational surfaces particles which entered the microwave beam once re-enter it a few times after making a small number of toroidal revolutions (3 in this case). In the nonlinear case, the finite distortion of the distribution function in the velocity space region occupied by these parti-

cles is formed immediately after the first passing of resonant particles through the beam and is weakly modified by collisions in between the re-entries. This distortion (plateau formation) leads to a reduction of power absorption and current generation. Such a result cannot be reproduced by the quasilinear modeling where a bounce-averaged Fokker-Planck code is used and where, in addition, averaging over the flux surface is also applied to the kinetic equation (thus removing any dependence on the safety factor). In fact, within the Fokker-Planck modeling, the distortion of the distribution function results from the slow accumulation of small changes after many passings of a particle in the wave beam. Fig. 4 also shows that with increasing beam width (i.e., increasing nonlinearity) the region of reduced absorbed power and driven current tends to broaden.

4. Conclusions

A numerical model for ECCD in a tokamak which takes into account nonlinear wave-particle interaction has been developed. The modeling of ECCD for the 2nd harmonic X-mode shows that, for ASDEX Upgrade parameters, the broadening of the power deposition profile and of the current density profile appears with increasing beam width. Wide beams (≈ 10 cm) are to be expected in ITER, therefore, nonlinear effects might be important there. Moreover, the scaling of the nonlinearity parameter, ϵ_{NL} , (see above) indicates that the increase of the main magnetic field, B_0 , and of the input power, P , in ITER as compared to ASDEX Upgrade will contribute to enhance the role of nonlinear effects there.

In stellarators the convective transport of supra-thermal electrons plays a significant role. In Ref. [5] it has been shown that convective fluxes in stellarators are very sensitive to the detailed structure of the supra-thermal particle source. In the present study it is shown that, in high power ECRH/ECCD using the 2nd harmonic X-mode, nonlinear effects of wave-particle interaction are dominant in the formation of such a source.

It has been found that, during ECCD with 2nd harmonic X-mode, power absorption and current generation are sensitive to rational magnetic surfaces. This peculiarity might be important and useful for neoclassical tearing mode stabilization in tokamaks.

References

- [1] R. Kamendje, S. V. Kasilov, W. Kernbichler and M. F. Heyn. *Physics of Plasmas*, **10**, 75 (2003).
- [2] E. Poli, A. G. Peeters and G. V. Pereverzev. *Comp. Phys. Comm.*, **136**, 90 (2001).
- [3] T. Antonsen and K. Chu. *Physics of Fluids*, **25**, 1295 (1982).
- [4] S. V. Kasilov and W. Kernbichler. *Physics of Plasmas*, **3**, 4115 (1996).
- [5] S. V. Kasilov, W. Kernbichler, V. V. Nemov and M. F. Heyn. *Physics of Plasmas*, **9**, 3508 (2002).

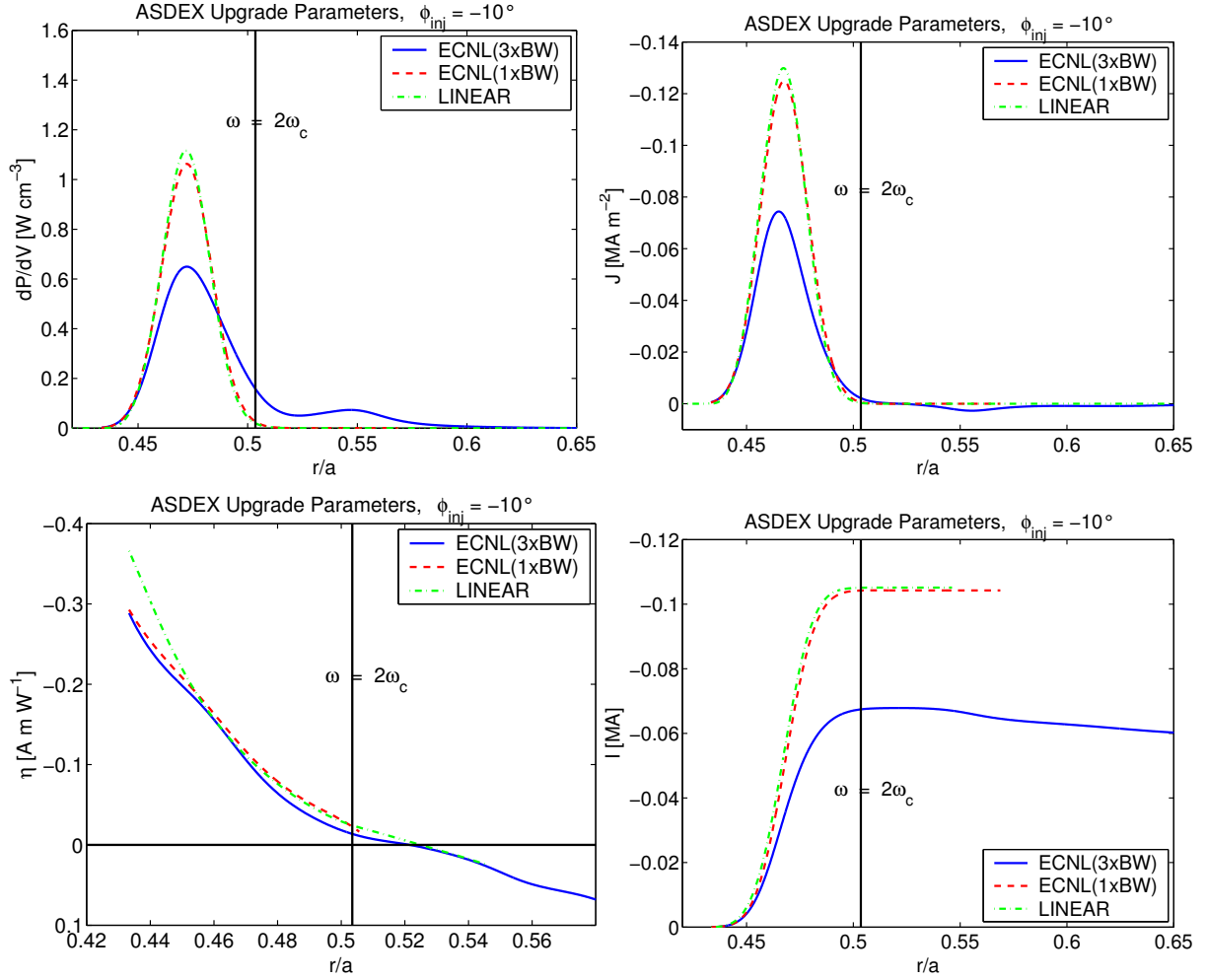


Figure 1: Profiles of the absorbed power density, dP/dV , of the parallel current density, j_{\parallel} , of the current drive efficiency, η , and of the total driven current, I , as obtained within linear and nonlinear computations. The indication 3xBW means that the parallel beam width has been increased 3 times.

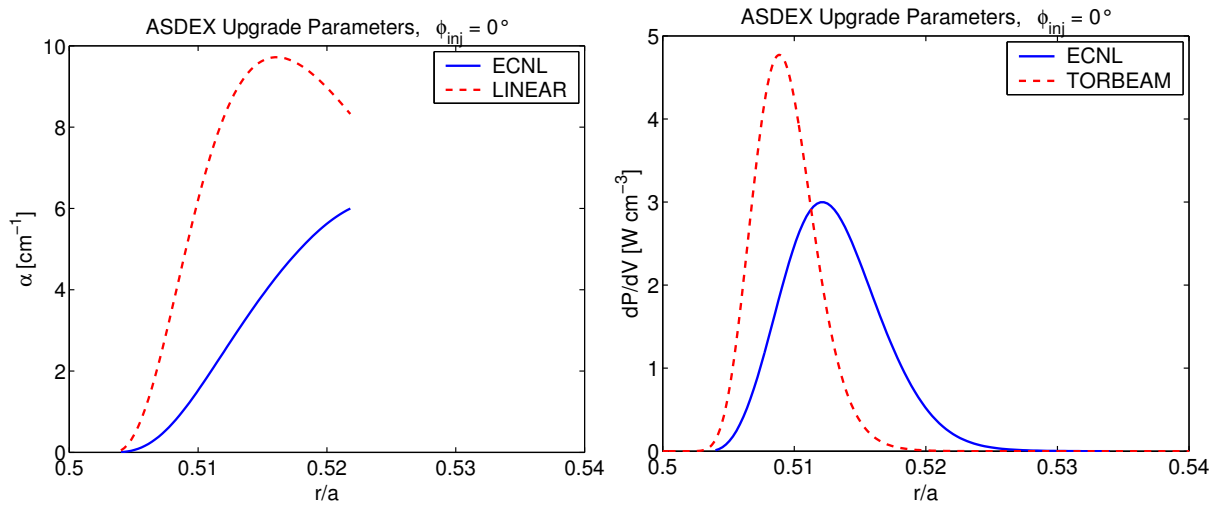


Figure 2: Profiles of the absorption coefficient, α , and of the absorbed power density, dP/dV , as obtained within linear and nonlinear computations for the case of perpendicular launch.

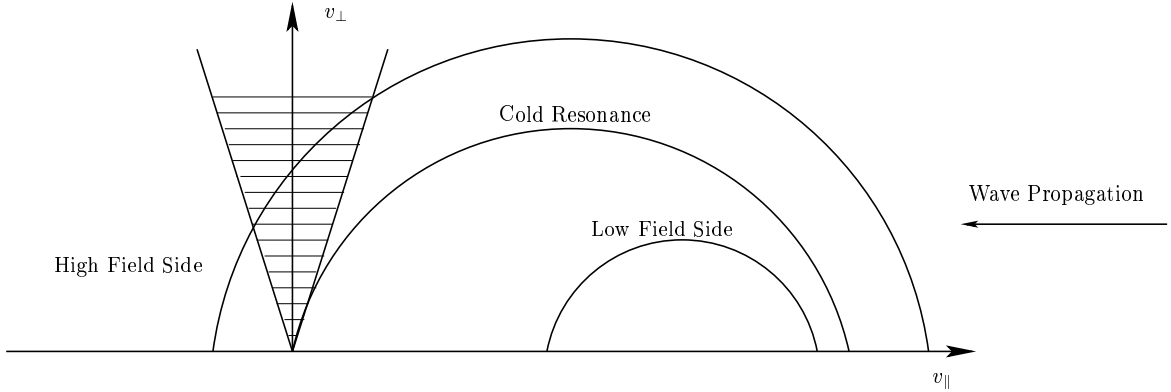


Figure 3: Respective locations of the resonance curves in velocity space for absorption on the low field side, at the cold resonance position and on the high field side. Low and high field side here are defined with respect to the position of the cold resonance surface $\omega = 2\omega_c$. The cone indicates the region where nonlinear effects of wave-particle interaction are important. Note that in the present case the wave beam is launched from the low field side.

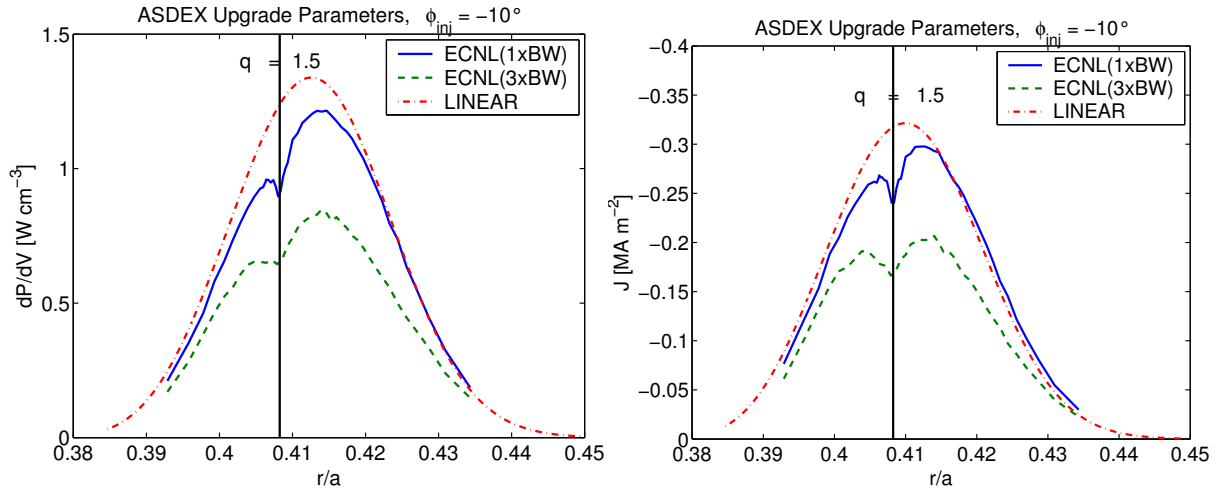


Figure 4: Profiles of the absorbed power density, dP/dV and of the parallel current density, $j_{||}$ as obtained within linear and nonlinear computations for a deposition around the rational surface $q = 3/2$. The indication 3xBW means that the parallel beam width has been increased 3 times.