Neoclassical Plateau Regime Transport in a Tokamak Pedestal

Istvan Pusztai[1], Peter J. Catto[2]

[1] Nuclear Engineering, Chalmers University of Technology and Euratom-VR Association, Göteborg, Sweden

[2] Plasma Science and Fusion Center, Massachusetts Institute of Technology, Cambridge, MA 02139, USA

CHALMERS

In tokamak pedestals the radial scale of plasma profiles can be comparable to the ion poloidal Larmor radius, thereby making the radial electrostatic field so strong that the $\mathbf{E}\times\mathbf{B}$ drift has to be retained in the ion kinetic equation in the same order as the parallel streaming. The modifications of neoclassical plateau regime transport – such as the ion heat flux, and the poloidal ion and impurity flows – are evaluated in the presence of a strong radial electric field [1]. The altered poloidal ion flow is most pronounced in the region of the strongest radial electric field where it modifies the friction of the electrons with the ions and can lead to an increase in the bootstrap current, by enhancing the coefficient of the ion temperature gradient term.

Motivation

- Tokamak pedestal $L_n \sim \rho_{\rm pol}$
- Subsonic flows $\partial_\Psi \ln n_i \approx -(Ze/T_i)\partial_\Psi \Phi$
- Radial electrostatic field ~ 100 kV/m [2]
- The contribution of the $\mathbf{E}\times\mathbf{B}$ drift to the poloidal motion can be comparable to that of the parallel streaming. They have to be retained in the same order in the ion kinetic equation.
- Trapped particles: $(\mathbf{v}_\parallel + \mathbf{v}_{\mathbf{E}\times\mathbf{B}})\cdot\nabla\theta \approx 0$ instead of $v_\parallel \approx 0$
- The normalized electric field $U = v_{\mathbf{E}\times\mathbf{B}} B/(v_i B_p)$ can be order unity
- The pedestal can be in the plateau collisionality regime $\epsilon^{1/2} \ll \nu_i qR/v_i \ll 1$ (for example Alcator C-Mod [2])

Ion transport and parallel ion flow

- We assume a quadratic electric potential well.
- The canonical angular momentum $\Psi_* = \Psi - \frac{Mc}{Ze}R\mathbf{v}\cdot\hat{\zeta}$ is used as the radial coordinate instead of Ψ [3], and $\mathcal{E} = E - Ze\Phi(\Psi_*)/M$ as an energy variable (conserved by the Vlasov operator).
- $\dot\theta = (v_\parallel \mathbf{b} + \mathbf{v}_{\mathbf{E}\times\mathbf{B}})\cdot\nabla\theta = (v_\parallel + u)\mathbf{b}\cdot\nabla\theta$, where $u = cI\Phi'/B$.
- We solve for the resonant part of the distribution function H_i defined in terms of the gyro-averaged perturbed distribution

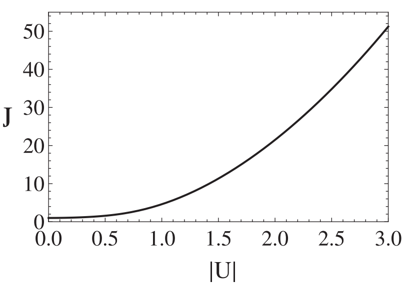
$$\bar f_{1i} = H_i + \frac{MBkv_\parallel f_{Mi}}{T_i} - \frac{Iv_\parallel f_{Mi}}{\Omega_i}\left(\frac{\partial \ln p_i}{\partial\Psi} + \frac{Ze}{T_i}\frac{\partial\Phi}{\partial\Psi}\right),$$

- When $H_i(\Psi_*)$ is transformed back to flux surfaces, the orbit squeezing factor drops out.
- $C^l_{ii}\{H_i\} \to -\nu H_i$, since the form of the collision operator acting on the resonant part H_i cannot matter in the plateau regime. The momentum conserving property of the operator is restored by $H_i \to H_i + MBkv_\parallel f_{Mi}/T_i$.
- The unknown k is determined from the ambipolarity condition

$$0 \approx \langle \mathbf{\Gamma}_i\cdot\nabla\Psi\rangle \approx -\sqrt{\frac{\pi}{2}}\frac{I^2\epsilon^2 n_i}{\Omega_i^2 qR_0}\left(\frac{T_i}{M}\right)^{3/2} e^{-U^2}$$
$$\times\left\{\left(\frac{1}{2} + 2U^2 + 3U^4 + 6U^6\right)\frac{\partial \ln T_i}{\partial\Psi} + \left[1 + 2\left(U^2+U^4\right)\right]\frac{Zek\langle B^2\rangle}{IT_i c}\right\}.$$

$\Rightarrow k = -\frac{J(U^2)}{2}\frac{\partial \ln T_i}{\partial\Psi}\frac{IT_i c}{Ze\langle B^2\rangle}$ (valid below $U \approx 3.5$) with

$$J(U^2) = \frac{1 + 4U^2 + 6U^4 + 12U^6}{1 + 2\left(U^2+U^4\right)}.$$

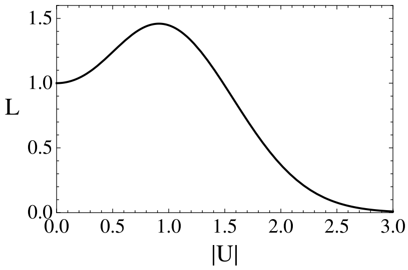


- The **ion heat flux** is given by

$$\langle \mathbf{q}_i\cdot\nabla\Psi\rangle = -3\sqrt{\frac{\pi}{2}}\frac{I^2\epsilon^2 p_i}{\Omega_i^2 qR_0}\left(\frac{T_i}{M}\right)^{3/2}\frac{\partial \ln T_i}{\partial\Psi}L(U^2),$$

with

$$L(U^2) = e^{-U^2}\frac{1 + 4\left\{U^2 + 2U^4 + \left[(4U^6 + U^8)/3\right]\right\}}{1 + 2(U^2 + U^4)}.$$



- To calculate the bootstrap current the **ion parallel flow** needs to be evaluated

$$n_i V_{\parallel i} = \int d^3v v_\parallel \bar f_{1i} \approx -\frac{Ip_i}{M\Omega_i}\left[\frac{\partial \ln p_i}{\partial\Psi} + \frac{Ze}{T_i}\frac{\partial\Phi}{\partial\Psi} + \frac{J(U^2)}{2}\frac{B^2}{\langle B^2\rangle}\frac{\partial \ln T_i}{\partial\Psi}\right].$$

- Using radial pressure along with $\langle BV_{\parallel i}\rangle = \langle BV_{\parallel Z}\rangle$ [4], we obtain the **poloidal flow of a collisional trace impurity**

$$V_{Z,\theta} = \frac{cIT_iB_\theta}{eZ_i\langle B^2\rangle}\left[\frac{T_zZ_i}{T_iZ_Z}\frac{\partial \ln p_Z}{\partial\Psi} - \frac{\partial \ln p_i}{\partial\Psi} - \frac{J(U^2)}{2}\frac{\partial \ln T_i}{\partial\Psi}\right].$$

Bootstrap Current

- Electron-ion collisions depend on the ion mean flow, the electron distribution experiences this friction and is thereby influenced by the presence of the electric field.
- The **bootstrap current** is calculated using an adjoint method [5]

$$\langle j_{BS}B\rangle = -\sqrt{\frac{\pi}{2}}\frac{\epsilon^2 cIp_e v_e}{\nu_e qR_0}\frac{\sqrt{2}+4Z}{Z(2+\sqrt{2}Z)}$$
$$\times\left[\frac{1}{p_e}\frac{\partial p}{\partial\Psi} + \frac{\sqrt{2}+13Z}{2\left(\sqrt{2}+4Z\right)}\frac{1}{T_e}\frac{\partial T_e}{\partial\Psi} + \frac{J(U^2)}{2ZT_e}\frac{\partial T_i}{\partial\Psi}\right].$$

Discussion

- As the electric field increases, the resonance causing plateau transport, which would be at $v_\parallel \approx 0$ for $U = 0$, is now shifted towards the tail of the distribution. For strong electric field $U \gg 1$ this leads to an exponential reduction of the ion heat flux. However, for moderate values of U the ion heat diffusivity is enhanced $L(|U| \approx 0.91) \approx 1.46$.
- The temperature gradient driven part of parallel ion flow is multiplied by factor $J(|U|)$ which is a monotonically increasing function approaching an asymptote of $6U^2 - 3$ as U^2 goes to infinity. The same factor appears also in the expressions for the poloidal impurity rotation and the bootstrap current multiplying the ion temperature gradient term.

References

[1] I. Pusztai and P. J. Catto, Plasma Phys. Control. Fusion **52**, 075016 (2010). [The corrected results are given here, an errata is being prepared]

[2] K. D. Marr et al., Plasma Phys. Control. Fusion **52**, 055010 (2010).

[3] G. Kagan and P. J. Catto, Plasma Phys. Control. Fusion **50**, 085010 (2008).

[4] P. Helander, Phys. Plasmas **8**, 4700 (2001).

[5] T. M. Antonsen and K. R. Chu, Phys. Fluids **25**, 1295 (1982).

This work was funded by the European Communities under Association Contract between EURATOM and Vetenskapsrådet, and by U.S. Department of Energy Grant No. DE-FG02-91ER-54109 at the PSFC, MIT. We are also grateful for the economic support from the Adlerbert Research Foundation.