Parallel dynamics during transient filamentation in ELMs

Florin Spineanu, Madalina Vlad

Association EURATOM-MEdC Romania

Abstract

It has been shown that the layer evolves to the break-up of the layer with strong concentration of the density, vorticity and current density into cuasi-singular structures (filaments). This evolution is purely growing in an axisymmetric geometry and is governed by an universal instability of the same type as the Chaplygin gas with anomalous polytropic. The break-up of the current layer and formation of filaments takes place on very short time scales, governed by the Alfven speed in the direction of the layer (transversal on $\mathbf{B}$). This is a fast transient process which in axisymmetry is only limited by the generation of the singular filaments. The parallel dynamics is arbitrary.

However the parallel dynamics is inhibited by the same mechanism which damps the poloidal rotation in tokamak, the *magnetic pumping*. The fast increase of the current and of the flow velocity

along the magnetic field lines during the filamentation is accompanied by radial currents (localised to the filaments) due to curvature drifts which, in a collisional plasma, absorb energy from the flow on a time scale of the inverse ion-ion collision frequency. As in the usual neoclassical damping the effect is nonuniform along the line and is effectively equivalent to a force acting against the increase of the flow in the filaments. This may generate oscillatory instead of purely growing states. The effect of thermal channeling due to filamentation contributes to the reduction of the efficiency of the damping mechanism and the regime can again be purely growing but saturates the filaments a finite amplitude.

Vortex nucleation in strongly sheared velocity layers

Basic facts (in general supported by observations, still to be verified by experiments)

- in H mode a layer of plasma at the edge rotates poloidally with strong radial shear: this means a vorticity sheet
- there is a current sheet superposed on the vorticity sheet
- the current-vorticity sheet is unstable and breaks up into filaments

The mechanism of filamentation has the same nature as the instability of anomalous polytropic (Chaplygin) gases. The parallel dynamics is essential (is not collisional or Landau saturation).

Previous works: Ott, Trubnikov, Bulanov and Sasorov.

There is a sort of current sheet at the edge superposed on the sheared velocity layer

056121-6 Burrell *et al.*

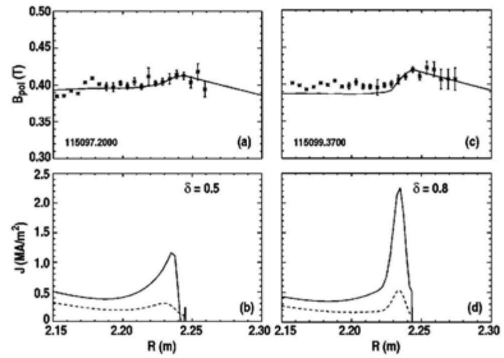


FIG. 9. Comparison of calculated poloidal magnetic field determined from the computed edge current density with the poloidal field measured by Zeeman polarization measurement from an injected lithium beam. (a) and (b) are for an upper single null discharge with triangularity 0.5 while (c) and (d) are for a balanced double null discharge with triangularity 0.8. The shapes are essentially identical to those shown in Fig. 7. In (b) and (d), the lower, dashed curve is the Ohmic plus bootstrap contribution to the overall current density while the upper, continuous curve is the total current density. Note that the current density is the local current density at the outer midplane of the plasma.

There is a current sheet

superposed on the sheared layer

Breaking into localised structures of the density distribution in a layer of current

The width of the layer is initially L_0 and it evolves to a profile L which is variable along the direction y of the layer (poloidal).

Unperturbed state: $A = A_z(x) = -LB_0 \ln\cosh\left(\frac{x}{L}\right)$.

Using the notation

$$\rho(t,y) = \frac{nL(t,y)}{n_0 L_0}$$

we have the usual density conservation

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial y}(\rho v) = 0$$

The current density is

$$j_z = en(v_{iz} - v_{ez})$$

where

$$v_{iz} - v_{ez} = \frac{cB_0}{2\pi e n L\left(t, y\right)} = \frac{\text{const}}{nL}$$

The equation of motion is

$$\begin{aligned} \frac{\partial v}{\partial t} + v\frac{\partial v}{\partial y} &= \frac{1}{n m_i c}\left(-j_z B_x\right) \\ &= \frac{e}{m_i c}\left(v_{iz} - v_{ez}\right)\frac{\partial A}{\partial y} \end{aligned}$$

When the system is invariant along the z direction then the generalized momenta of the electrons and of ions are conserved

$$\begin{aligned} m_i v_{iz} + \frac{e}{c}A &= \text{const} \\ m_e v_{ez} - \frac{e}{c}A &= \text{const}' \end{aligned}$$

Then the equation of motion is

$$\frac{\partial v}{\partial t}+v\frac{\partial v}{\partial y}=\frac{e}{m_i c}\left(v_{iz}-v_{ez}\right)\frac{\partial A}{\partial y}=c_0^2\frac{1}{\rho^3}\frac{\partial \rho}{\partial y} \tag{1}$$

The solution of the equations of the “Chaplygin” fluid

The two equations

$$\frac{\partial \rho}{\partial t}+\frac{\partial}{\partial y}\left(\rho v\right)=0$$

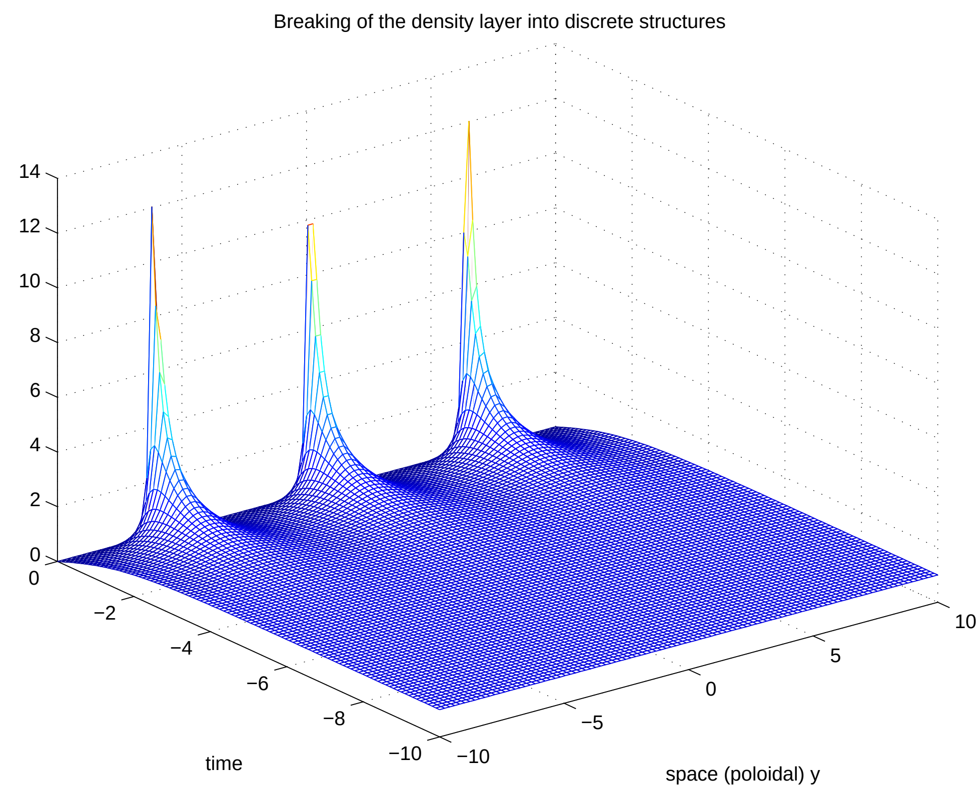
and

$$\frac{\partial v}{\partial t}+v\frac{\partial v}{\partial y}=c_0^2\frac{1}{\rho^3}\frac{\partial \rho}{\partial y}$$

is obtained using a *hodograph* transformation. The formulas are

$$\begin{aligned}\frac{nL}{n_0 L_0} &= \rho\left(t,y\right)=\frac{\sinh\left(|\tau|\right)}{\cosh\left(\tau\right)-\cos\chi}\\ \frac{v}{c_0} &= -\frac{\sin\chi}{\sinh\left(|\tau|\right)}\end{aligned}$$

Solution showing breaking into singular structure



Tearing of the current layer

The equations are

$$
\begin{aligned}
\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial x} &= -\frac{e}{m_i c}\left(V_z^{(0)} + \frac{e}{m_e c}A\right)\frac{\partial A}{\partial x} \\
\frac{\partial n}{\partial t} + \frac{\partial}{\partial x}(nv) &= 0 \\
\frac{\partial^2 A}{\partial x^2} + \frac{\partial^2 A}{\partial y^2} &= \frac{4\pi e L}{c}\delta(y)\, n\left(V_z^{(0)} + \frac{e}{m_e c}A\right)
\end{aligned}
$$

where

$$V_z^{(0)} \equiv v_{ez}^{(0)} - v_{iz}^{(0)}$$

The solution from Troubnikov

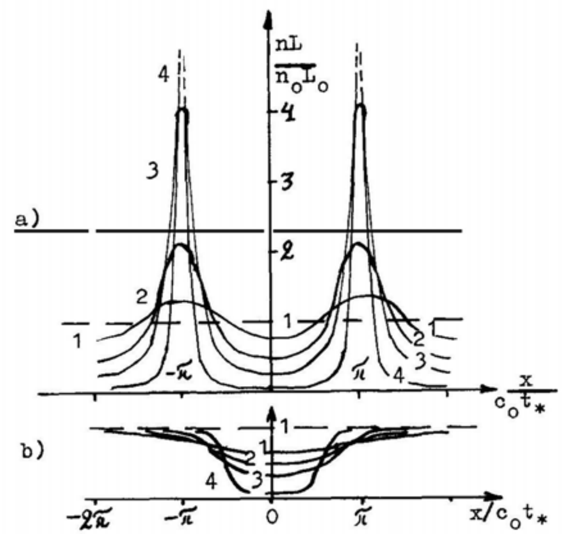


Fig. 12. Tearing of the plasma density (nL/n_0L_0) – periodic in "x" (a) and localized (b) – in a thin neutral current-carrying layer. Curves 1–4 correspond to the parameter $\gamma t = -2, -1, -0.5, -0.1$.

There is a breaking up of the density in the layer

The density is higher on both sides of the maxima of vorticity.

The parallel dynamics

The variables are Ω (vorticity) and $J_{\|}$. These are the equations:

$$
\begin{aligned}
\frac{d\omega_{\|}}{dt} &= \frac{B^2}{\rho}\nabla_{\|}\left(\frac{j_{\|}}{B}\right) - \frac{2\mathbf{B}\cdot\left(\nabla p \times \nabla B\right)}{\rho B^2} + \frac{\mu}{\rho}\nabla^2\omega_{\|} \\
\frac{\partial j_{\|}}{\partial t} &= \frac{1}{\mu_0}\nabla_{\|}\left(B\omega_{\|}\right) + \frac{\eta}{\mu_0}\nabla^2 j_{\|}
\end{aligned}
$$

The electrons and ions moving along a magnetic line experience different deviations induced by the gradient of B and by the curvature (guiding centre drifts). The charge separation is a current with nonvanishing radial component and symmetric periodic amplitude along the line. This current (transversal on the line) in an ideal plasma is simply oscillatory, following the regions of low or high B. The collisionality, even small, opposes a resistivity to this current density and induces an irreversible loss of energy, leading in time to the damping of the poloidal flow component, the one that is confronted with the variation of B.

The current density induced by the drift separation

$$j_D = en\,(v_{Di} - v_{De})$$

leads to a loss of energy $\Delta\mathcal{E} \sim \eta_\perp \left\langle j_D^2 \right\rangle t$ averaged over the excursion of the particles that explore the low-B and the high-B regions along the line in the time t. The average loss of energy per unit of time $\partial\left(\Delta\mathcal{E}\right)/\partial t$ is noted ε and its time integral is bounded from above by the total "poloidal" energy of the flow. An estimation is

$$j_D \sim en\frac{1}{\Omega_i}\left(\mu_i\nabla B + v_{\parallel i}^2\left(\widehat{\mathbf{n}}\cdot\nabla\right)\widehat{\mathbf{n}}\right) - en\frac{1}{\Omega_e}\left(\mu_e\nabla B + v_{\parallel e}^2\left(\widehat{\mathbf{n}}\cdot\nabla\right)\widehat{\mathbf{n}}\right)$$

We can neglect the ion field-curvature term due to smaller $v_{\parallel i}$ and the electron ∇B drift

$$j_D \sim en\frac{\mu_i\nabla B}{\Omega_i} - env_{\parallel e}^2\frac{1}{\Omega_e R}$$

from which we get an estimation of the power loss

$$\varepsilon \sim e^2n^2\eta_\perp\left(\frac{1}{\Omega_i}\left\langle\mu_i\nabla B\right\rangle - v_{\parallel e}^2\frac{1}{\Omega_e R}\right)^2$$

and of the maximal variation of the electron parallel velocity

$$v_{\parallel e} \sim \sqrt{\Omega_e R} \left| \frac{\varepsilon^{1/2}}{en\eta_\perp^{1/2}} - \frac{1}{\Omega_i} \langle \mu_i \nabla B \rangle \right|^{1/2}$$

The effect on the variation of the vorticity by stretching during filamentation is

$$\frac{d\omega_\parallel}{dt} = \frac{B^2}{\rho} \nabla_\parallel \left(\frac{j_\parallel}{B} \right)$$

Taking $\omega_\parallel \sim v/d$ where v is in the layer (poloidal) and d is the width of the layer,

$$\left. \frac{\partial v}{\partial t} \right|^{damping} = \frac{dB}{ne\rho} \frac{\partial}{\partial z} \sqrt{\Omega_e R} \left| \frac{\varepsilon^{1/2}}{en\eta_\perp^{1/2}} - \frac{1}{\Omega_i} \langle \mu_i \nabla B \rangle \right|^{1/2}$$

This new term must be added to the equation for the velocity v in the layer, and acts like a force opposing the concentration of current density in a filament.

$$\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial x} = -\frac{e}{m_i c}\left(V_z^{(0)} + \frac{e}{m_e c}A\right)\frac{\partial A}{\partial x} - \left.\frac{\partial v}{\partial t}\right|^{damping}$$

The most important effect is the inhibition of the Chaplygin (drop-on-ceil) instability.

Conclusions

A layer of plasma where there is vorticity concentration (sheared poloidal rotation) is also an attractor for current density concentration. This layer is unstable to separation into distinct highly concentrated regions which at the limit may become cuasi-singular. The nonlinear tearing instability driven by a dynamics similar to the anomalous polytropic (Chaplygin) gases can be inhibited by the parallel dynamics during the filamentation, governed by the magnetic pumping mechanism. The vorticity stretching and fast parallel variation of the parallel current are constrained by the magnetic pumping that places a limit. It must be examined if this limit generate a reactive mechanism to explain oscillatory behavior instead of purely growing filamentation.